



Research Article

Lattice Energy Converter

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Abstract

Multiple implementations of a Lattice Energy Converter (LEC) have demonstrated the ability to self-initiate and self-sustain the production of a voltage and current over extended periods of time. A LEC converts the internal energy within the lattice of some materials, such as palladium, or of gases occluded within the lattice, such as hydrogen or deuterium, into ionizing radiation and electrical energy. Experiments include tests where the current-voltage (I-V) characteristics of the LEC were measured when an external voltage/current was applied, as well as other I-V tests where the spontaneous LEC voltage was measured as a function of temperature and resistance. LEC voltage and current has been shown to increase with increased temperature. The electrical power produced by a LEC is similar to that produced by a nuclear battery however, a LEC does not require radioactive materials. While the energy levels produced to date are several orders of magnitude below those required for most power sources, the calculated flux of ionizing radiation necessary to produce the experimentally measured voltage and current would require the equivalent of several curies of radiation. These results have been independently replicated by two individuals. A video of the Lattice Energy Converter presentation, from the 2021 LENR workshop in honor of Dr. Srinivasan, is available at: https://www.youtube.com/watch?v=J4dzTWY_aWM This paper expands on the YouTube video presentation with additional analysis that supports the observed experimental results.

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1. Introduction

A Lattice Energy Converter (LEC) is a direct energy conversion device that converts the thermal energy of lattice vibration into ionizing radiation and electrical energy. The direct conversion of heat into electricity without the use of radioactive materials or mechanical means is a challenging yet promising method for the production of electrical energy. Figure 1 shows a Pd-H LEC cell that self-initiates and self-sustains the spontaneous production of a voltage and current into a load impedance. In its simplest implementation a LEC achieves direct conversion using a pair of electrodes separated by a gas at normal temperature and pressure (NTP). One electrode, designated the working

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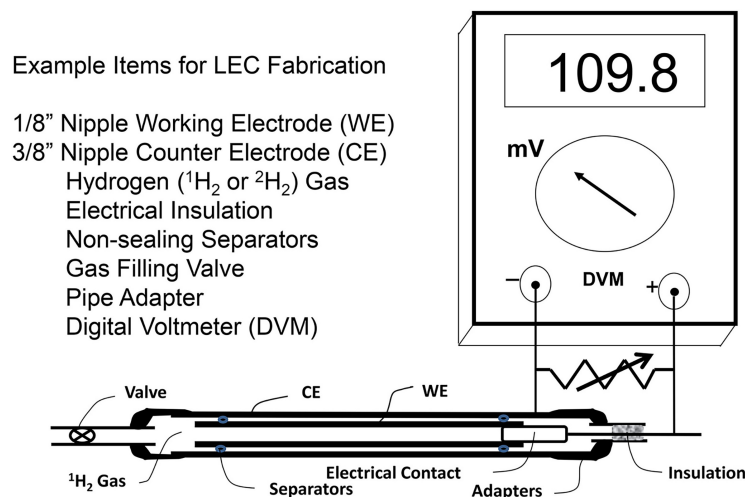


Figure 1. Self-initiating and self-sustaining Pd-H LEC cell under load.

electrode (WE), is comprised in part of a hydrogen host material, such as iron (Fe), nickel (Ni), or palladium (Pd), occluded with hydrogen or deuterium gas. The other electrode, designated the counter electrode (CE), may be a common metal, such as copper (Cu), zinc (Zn), or brass an alloy of Cu and Zn.

Experiments have established that when hydrogen host material is electrodeposited from an aqueous solution the WE will become ‘active’ and produce ionizing radiation. WE activity can be verified in air by placing the WE in close proximity to a CE and connecting a digital voltmeter (DVM) between the two electrodes. If no spontaneous voltage is observed the WE is returned to the plating bath and more hydrogen host material is codeposited on the WE. Multiple LEC configurations have been tested including tests where an external electrical voltage was applied between the working electrode and the counter electrode as well as tests where the LEC self-initiates and self-sustains the production of a voltage and current in the absence of an applied external voltage and current.

2. Background

2.1. Hydrogen-Metal interactions

The metal-hydrogen system [1] and in particular the palladium-hydrogen system [2], [3] have been studied for more than 150 years since H. Sainte-Claire Deville and E Troost [4] reported that hydrogen diffused rapidly through homogeneous plates of fused iron (Fe) and platinum (Pt). These surprising results led Thomas Graham, Master of the Royal Mint, to conduct a similar series of experiments with palladium (Pd). Three years later, Graham [5] was the first to report the high rate at which hydrogen would diffuse through heated Pd. Additionally, Graham found that Pd could absorb over 600 times its volume of hydrogen. More recently, the diffusion of hydrogen in the palladium-hydrogen (Pd-H) system as well as the nickel-hydrogen (Ni-H) and iron-hydrogen (Fe-H) systems has been documented by Mehrer [6]. Fukai [1], when discussing the role that lattice vacancies and in particular superabundant vacancies (SAVs) have in determining the properties of materials, points out:

The real equilibrium phase diagrams including M-atoms vacancies have not been obtained so far. This recognition brought us to the expectation that SAVs should be formed in the process of electrodeposition of metals from aqueous solutions. There M and H atoms are deposited simultaneously, and in this process appropriate numbers of vacancies can be incorporated.

Experimentally, WE's have been produced by the electrodeposition of hydrogen host material from an aqueous PdCl_2 and LiCl solution as well as by the co-deposition of Pd from an aqueous solution of PdBr_2 without LiBr , which was used by J. P. Biberian [7] to produce an active WE and replicate the LEC results. WE's have also been produced by codeposition of iron from an aqueous solution of $\text{FeCl}_2 \cdot 4\text{H}_2\text{O}$, and other materials or alloys are expected to produce active WE's.

2.2. Electrical properties of gases

Similarly, the electrical properties of gases have been studied for more than 130 years. In 1896, J.J. Thomson and E Rutherford [8] published an important paper 'On the Passage of Electricity through Gases exposed to Röntgen (X-) Rays.' In the same year, Thomson [9] gave a series of lectures on *The Discharge of Electricity through Gases* at Princeton University in New Jersey. This was followed in 1899 [10] by his definitive paper 'On the theory of the conduction of electricity through gases by charged ions' where he developed a mathematical theory that would predict the ionization rate per unit volume of gas from a measurement of the current density per unit area of the conduction path. In 1903 he published the 1st edition of his treatise *Conduction of Electricity Through Gases* [11]. In 1906 he published the 2nd edition [12] of his treatise, the same year he was awarded the Nobel Prize in Physics

in recognition of the great merits of his theoretical and experimental investigations on the conduction of electricity by gases,

This was then followed in 1928 [13] and 1933 [14] by the expanded two volume 3rd edition coauthored with his son G.P. Thomson, soon to be Nobel Laureate (1937).

The theory of the conduction of electricity through gases can be used to analyze the performance of a LEC with the objective of both understanding the phenomenon and of optimizing its performance in order to realize implementations of practical application. Toward this goal the following works by J.S.E Townsend [15], K.K Darrow [16], and L.B. Loeb [17], are of particular interest. Townsend was a research assistant to J.J. Thomson and although he is remembered for his work on gas discharges, *i.e.*, the Townsend avalanche discharge, his discussion in his book [15] of 'The Motion of Ions in Gases' is instructive. Darrow was a Research Physicist at Bell Laboratories in New Jersey and his discussion of the diffusion of ions to the measurement of the current density per unit area in his book titled *Electrical Phenomena in Gases* is of particular importance and will be discussed in detail in a following section. Loeb was a professor of physics at the University of California at Berkeley for many years and his book titled *Basic Processes of Gaseous Electronics* [18] includes advances in understanding made during WWII.

The important observation from the above references is that there are several properties of the gas that need to be carefully considered in order to analyze the performance of a LEC: the rate of generation of the ions; the attachment of ionized electrons to neutral molecules; the recombination rate of ions back into neutral molecules; the density of ions in the gas; the mobility of the ions in the gas; the space charge within the gas due to the difference in the number of positive and negative ions; the electric field strength in the gas; the diffusion of ions due to concentration gradients in the gas; as well as, the current density produced per unit cross-sectional area of the conduction path which is cell geometry dependent.

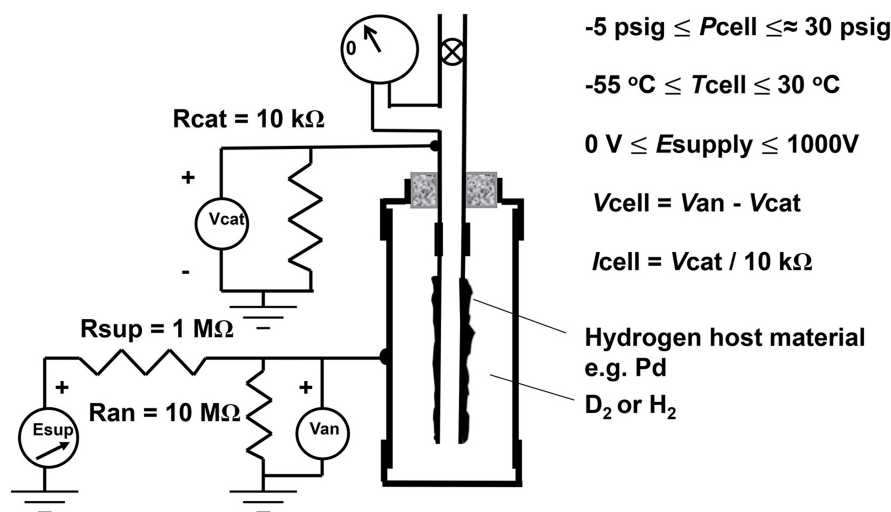


Figure 2. LEC cell configured with a variable external voltage supply to measure current vs. voltage (I-V) characteristic.

3. Experimental results

3.1. Experimental setup and tests with external variable voltage supply

Initial LEC experimentation began with the objective to conduct electrolysis in a gas in order to produce fugacity and thus electrically load hydrogen into a palladium working electrode at temperatures above 100 °C. Experimental evidence by multiple LENR scientists indicated that LENR output increased with increasing temperature and loading but experiments that use liquid electrolysis and are limited to temperatures below the boiling point of the liquid electrolyte.

Figure 2 illustrates the initial test cell that was designed with the objective to greatly increase the operating temperature and thus increase LENR output. This cell includes a working electrode composed of a 1/4 inch copper tube that had been codeposited with palladium from an aqueous solution of 0.03 M PdCl₂ and 0.3 molar LiCl. The working electrode was positioned inside a 3/4 inch brass pipe which served as the counter electrode and the cell was assembled using standard pipe fittings, including a non-conducting bushing to electrically isolate the working electrode from the counter electrode.

After assembly, a vacuum was pulled and the cell filled with deuterium gas. Since gas is a non-conductor unless it contains ions, six 1 μCi sources of Am-241 were placed in the cell to ionize the gas during initial tests. Also shown in Figure 2 is a high voltage DC power supply in series with a 1 M Ω current limiting resistor for personnel and instrumentation safety. A Labjack U6 Pro with capability to record up to 14 channels of data is connected to the working electrode (WE) and the counter electrode (CE) in order to measure the voltages to ground. A resistor voltage divider was placed in the CE circuit in parallel with the Labjack input to limit the voltage to no larger than ± 10 volts. Initial tests using this Pd-D LEC cell configuration produced more conduction through the gas, *i.e.*, more ions in the gas, than what 6 μCi of Am-241 would produce if there were no Pd codeposited on the WE. When the Am-241 sources were removed, the LEC cell still conducted a current.

A second important observation was that the LabJackTM instrumentation system with a sample rate of 512 samples per second detected voltage spikes and short-term excursions that occurred on time scales of less than 2 ms. Figure 3 is a screen shot showing the LabJack real-time display of the LEC amplitude. The sample rate used during this test

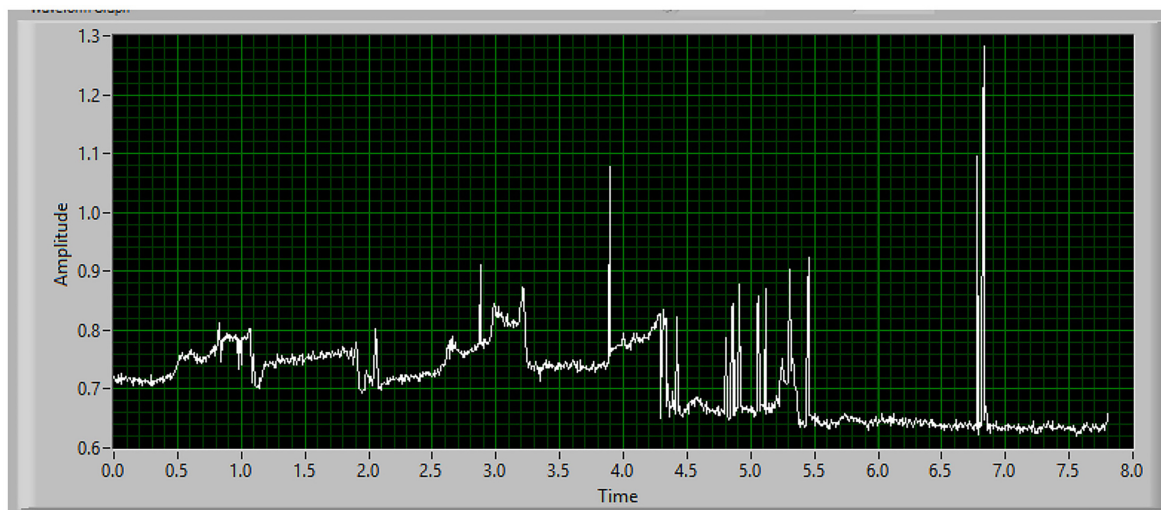


Figure 3. Typical real-time display of the LEC amplitude in volts with 8 seconds of data continuously scrolling across the screen.

was 128 samples per second. The vertical axis displays the Amplitude in volts and the horizontal axis displays Time in seconds. Since the LabJack instrumentation system is limited to ± 10 volts, a scale factor is used to correct the voltage in real-time during concurrent processing with data logging.

In order for data processing to keep up with data collection, software was developed to run concurrently with LabJack using a combination of Microsoft Excel and Visual Basic so that as soon as a file was recorded, it was processed and displayed. At a minimum, data processing for each channel includes: scaling the data; file averages; one second averages; and maximum and minimum voltages for each second. Additional processing has included the use of a Ludlum proton recoil scintillation detector (PRESCILA) to detect neutrons and sodium iodide detectors to detect gamma including energy levels, as well as signal processing of various environmental channels such as ambient test stand temperature, LabJack laboratory temperature, and the high-voltage power supply voltage. Typically, up to four LEC cells are simultaneously recorded and processed along with temperatures with the 14 channels available.

Multiple tests have been conducted to characterize the LEC performance such as changing the polarity of external voltage applied, changes in cell temperature and gas pressure, changes in cell dimensions, and changes in load resistance. LEC cells have also been tested using deuterium gas, hydrogen gas, and even atmospheric air which has approximately 0.5 parts per million of hydrogen. When exposed to air, the activity decayed over several hours which suggests that the activity was in part the result of outgassing of the hydrogen, but some activity was observed even after several days.

One example is shown in Figure 4 where the temperature in the cell was reduced to approximately minus 55 °C to reduce the number of water vapor ion clusters in the gas [19] and thereby eliminate them as the source of ions contributing to cell conduction. For this test, after the cell temperature had stabilized, the voltage was reduced in steps of 100 volts from a high of 800 volts down to 100 volts where the step size was reduced to 10 volts. Although the high-voltage supply was initially at 800 volts, the voltage across the LEC was approximately 275 volts due to cell conduction and the current limiting resistor. As the voltage is reduced, the current also is reduced and a current-voltage (I-V) characterization of the conduction is produced.

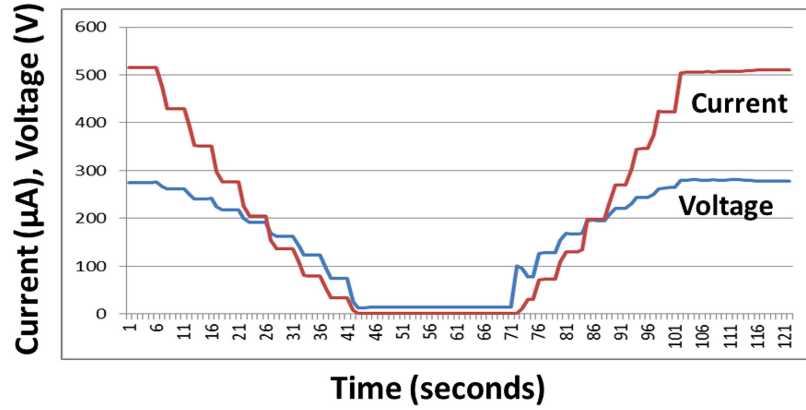


Figure 4. Measured I-V characteristic of a LEC cell tested at -55 °C conducted over a 2 minute time period.

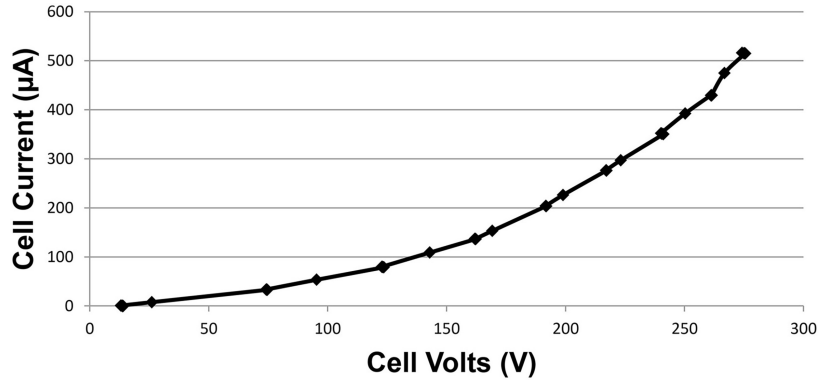


Figure 4. I-V relationship between current and voltage at -55°C.

The lower plot (fig. 4b) is current vs. voltage (I-V). Surprisingly however, on closer examination of the raw data it appears that the current goes to zero before the voltage goes to zero. Figure 5 is a semilog plot of the current vs. voltage (I-V) data which shows a change in the slope that started occurring at about 28 volts and the measured voltage does not go zero to as the current tends toward zero.

Figure 6 shows that as V approaches zero with slope $m \approx 0.0241$, the current, $I_e(V)$, does not extrapolate to zero as the term $I_e^{1/3}(0)^3 \approx 1.3$ remains. However, the Thomsons' [13] write $V(i)$ as a polynomial in current density i , i.e., $V = Ai^2 + Bi$. This is a significant difference since the LEC curve is convex up while the Thomsons' curve is concave down. This difference may be attributed to fugacity loading of the Pd with increased voltage and a commensurate increase in the ionization flux.

3.2. Experiments without external voltage supply

These surprising results could easily be checked by removing all external voltage and current from the LEC cell. As shown in Figure 1, to eliminate any possibility of a sneak circuit from the LabJack instrumentation system, the only

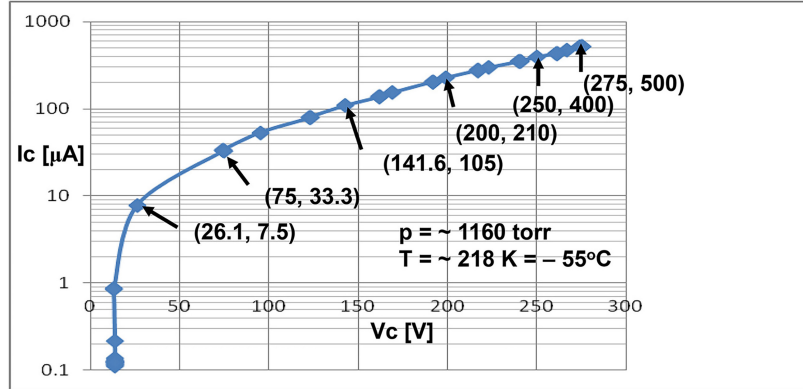


Figure 5. Semilog plot of data from the LEC cell test of figure 4 showing that the current goes to zero before the cell voltage goes to zero.

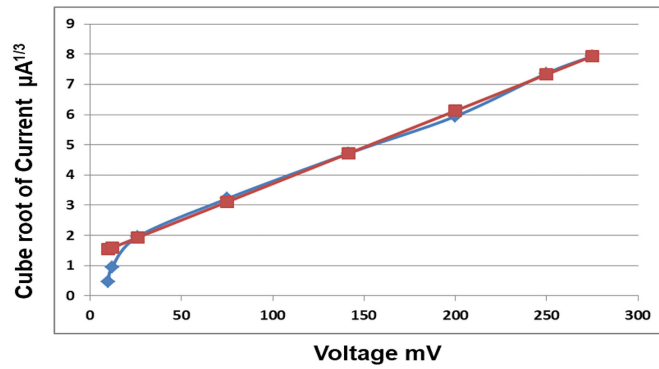


Figure 6. Plot of the cube root of the current vs the potential – Blue: Experimental data – Red: polynomial data fit: $I(V)^{1/3} = [mV + I_e^{1/3}(0)]$ where $m \approx 0.0241$ (33) and $I_e^{1/3}(0) \approx 1.3$.

instrumentation attached to the LEC was a DVM (Digital Voltmeter) with a $10\text{ M}\Omega$ internal impedance in parallel with a variable load resistor R resulting in an effective resistance of $936\text{ k}\Omega$.

When the codeposited Pd WE cell was assembled and filled with hydrogen, it self-initiated and self-sustained the production of both an ‘open circuit ($10\text{ M}\Omega$)’ a voltage and a ‘short-circuit’ ($680\text{ }\Omega$) current. To properly characterize a LEC cell both ‘open-circuit’ and ‘closed-circuit’ measurements are needed. An even better characterization can be made if multiple I - $V(R)$ values can be obtained. LEC output as a function of temperature is plotted in Figure 7, where the ‘open-circuit ($R_L \sim 936\text{ k}\Omega$)’ voltage increased from approximately $10\text{ }\mu\text{V}$ at $28\text{ }^\circ\text{C}$ to more than 525 mV at a temperature of $185\text{ }^\circ\text{C}$, an increase in voltage of over 50,000:1.

Self-sustaining LECs such as shown in Figure 1 are load tested by changing the variable load resistor that is in parallel with the DVM as shown in Figure 1. The variable R resistor box has 24 resistance values ranging from $1\text{ M}\Omega$ down to $10\text{ }\Omega$ where the voltage produced at each resistance used, about 17 descending resistance values, is plotted in Figure 8.

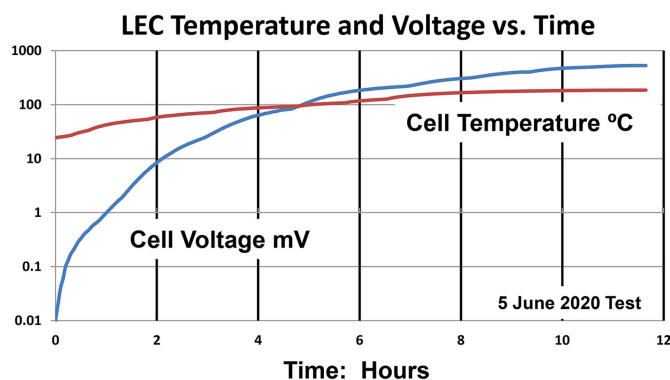


Figure 7. Plot of LEC cell temperature and ‘open-circuit ($\sim 936 \text{ k}\Omega$)’ voltage vs. time.

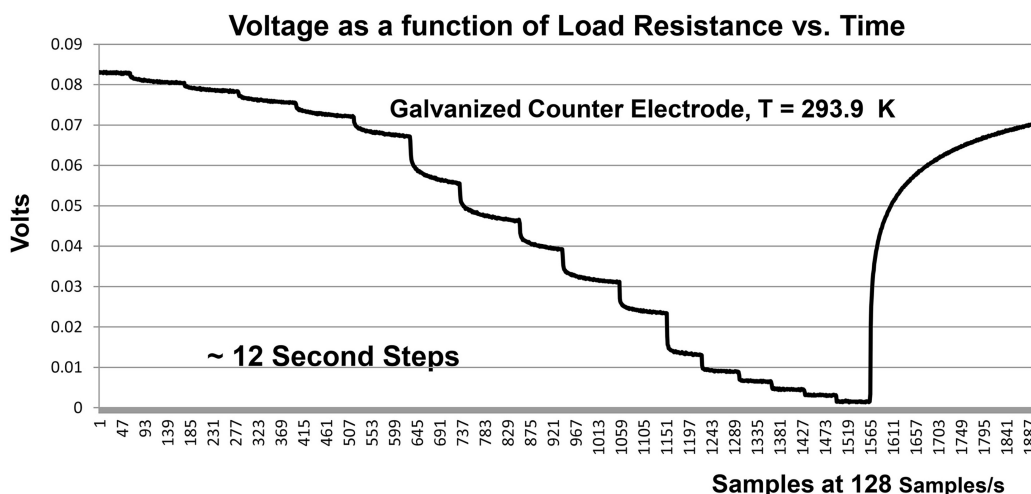


Figure 8. Plot of LEC cell voltage as a function of resistance at room temperature.

The resistance load was changed at approximately 12 second intervals in order to avoid deloading hydrogen from the palladium hydrogen-host-material. When the resistance value was switched from 680Ω back to $1 \text{ M}\Omega$, a time constant of voltage recovery is observed which may be related to the diffusion and mobility of hydrogen ions in the gas due to the change in electric field strength in the gas.

Figure 9 is a plot of LEC cell load testing of the cell performance shown in Figure 7 at three temperatures of 80, 140, and 185 °C. The three curves at the top of the figure plot the LEC voltage versus resistance for each temperature and the lower curves plot the calculated load current using Ohms law as well as the calculated shunt current that depends on cell voltage.

Of particular note is the relatively constant current observed for low values of load resistance when the voltage across the cell is small. Two possible mechanisms to produce a constant current include the direct charge method that can take place in a vacuum wherein particles or ions emitted by radioactive decay, thermionic emission, or the

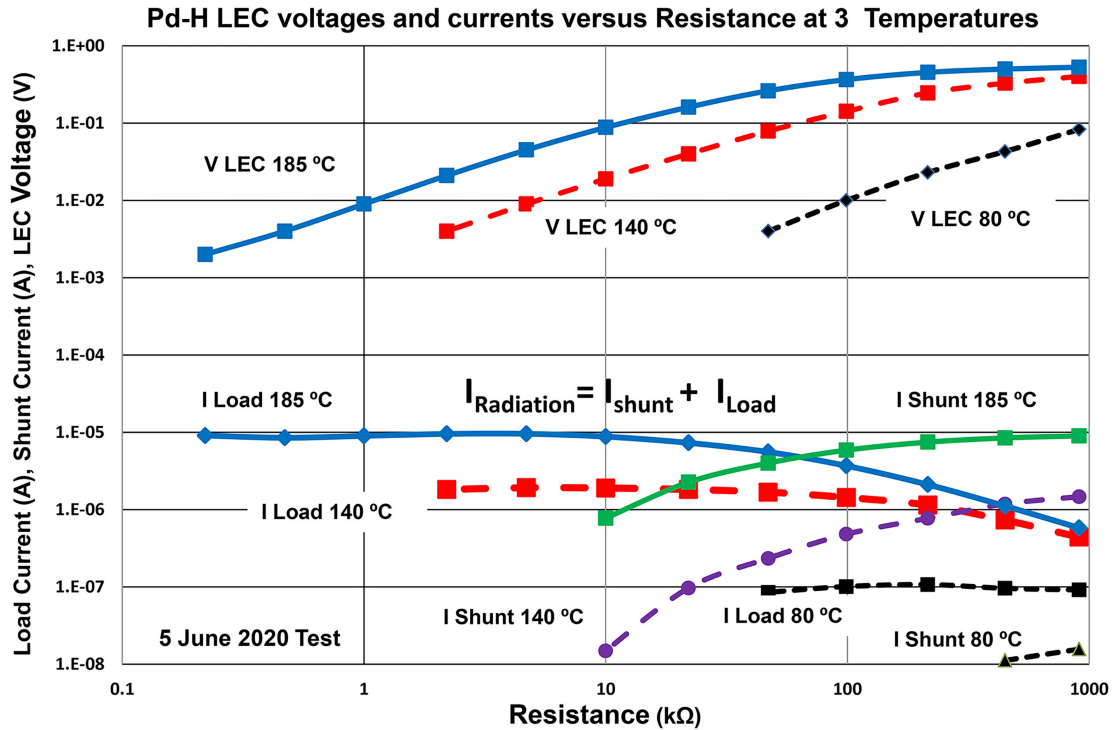


Figure 9. Plot of voltage and currents of a LEC cell as a function of load resistance at three different temperatures. See text for explanations of I_{Load} , I_{Shunt} , and $I_{Radiation}$.

photoelectric effect and the diffusion of ions in a gas due to a concentration gradient. This constant current behavior is in contrast to the predicted I-V behavior reported by Thomson and others at higher values of cell voltage. By neglecting diffusion in their analysis, they report that for small values of the voltage, the current increases linearly with voltage until asymptotically approaching a constant value that they called the saturation current at high values of voltage. This linear behavior at low voltage is observed for a LEC when calculating its shunt current. However, when evaluating LEC performance, care should be taken to collect voltage data for low resistance values to quantify the diffusion current conduction behavior of LEC cells.

The power curves peak at the point where the LEC cell internal impedance matches the external load impedance which is predicted by electrical engineering theory and is typical of conventional power supplies. Of note, the open-circuit voltage and the short-circuit current of a LEC which does not require radioactive material is comparable to a commercial P100 NanoTritiumTM nuclear battery that uses 225 mCi of radiation.

Figure 11 illustrates a possible combined phenomenological, physical, and electrical representation to explain the electrical behavior of the experimental palladium-hydrogen (Pd-H), palladium-deuterium (Pd-D), and iron-hydrogen (Fe-H) LEC cells that have been measured experimentally.

This is similar to a Norton equivalent circuit of a two-terminal linear electrical device where the current source and the cell impedance are the Norton circuit elements, however, in this LEC cell equivalent circuit the impedance Z_C depends upon the voltage. Inspection of the LEC cell's current behavior indicates that the cells behave as if the observed current originates from a current source. Thus, in the schematic representation, the cell's radiation generated

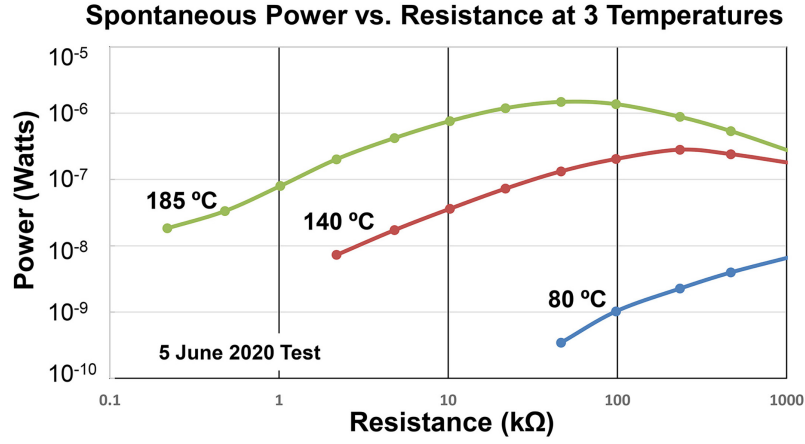


Figure 10. Plots the LEC cell load power as a function of load resistance at the three temperature of the cell in figure 9.

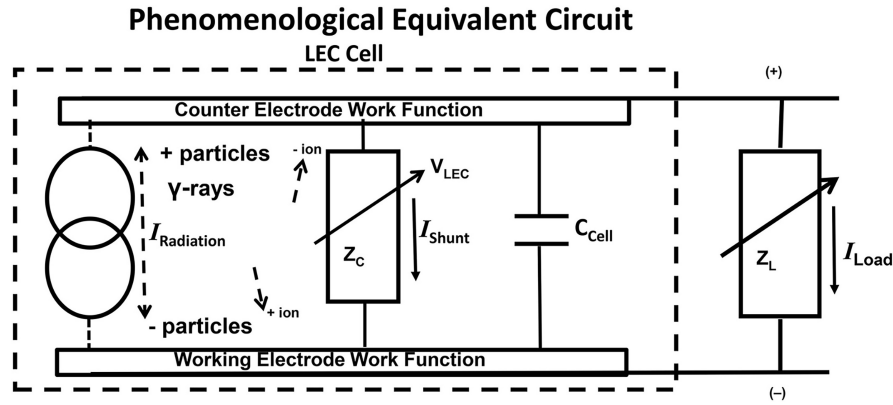


Figure 11. Combined phenomenological and physical schematic based on a Norton equivalent circuit.

gradient of ion density is shown as a current source. The working electrode and counter electrode are shown as having work functions (WFs) that might possibly be different for the working electrode and the counter electrode due to the different materials that might be used for these electrodes. For the LEC cell the hydrogen ($^1\text{H}_2$) or deuterium ($^2\text{H}_2$) gas and its associated ions also are indicated. The internal ion-ion plasma conduction current I_{Shunt} and cell voltage V_{LEC} characterize a variable voltage conduction impedance Z_C . The electrical capacitance of the cell is represented by C_{Cell} . The external variable load impedance is represented by Z_L and the load current I_{Load} is represented by an arrow.

The changes in the cell's load currents at higher cell voltages can be explained by an increasing internal cell current I_{Shunt} that shunts some of the thermally generated spontaneous radiation generated current, $I_{\text{Radiation}}$, away from the load impedance Z_L and through the voltage variable internal cell impedance Z_C . The changes in the cell's 'short-circuit' current and 'open-circuit' voltage capability and performance at higher temperatures can be explained by hypothesizing that $I_{\text{Radiation}} = I_0 \exp(-E_a/k_B T)$ where E_a is an activation energy associated with the WE, k_B is Boltzmann's

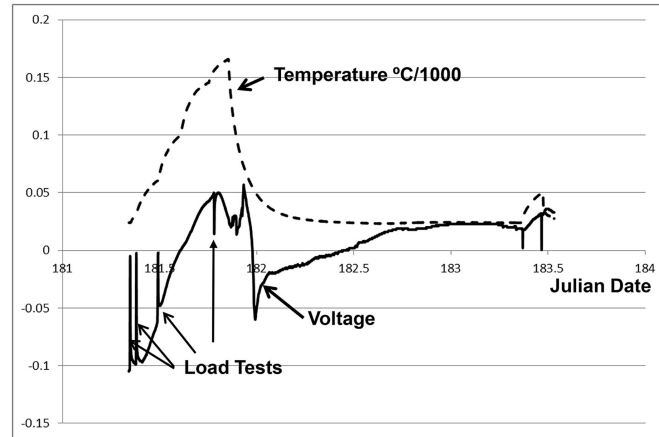


Figure 12. Example of complex spontaneous LEC cell behavior as a function of time and temperature.

constant, and $k_B T$ is the thermal energy associated with the hydrogen-host-material's lattice and its occluded hydrogen. This hypothesis is supported by the fact that when an Arrhenius plot, $\ln(I_{\text{Radiation}}) = -(E_a/k_B)(1/T) + \ln(I_0)$, of the radiation current at the three temperatures 80, 140 and 185 °C was made it resulted in a straight line of slope $-(E_a/k_B)$ resulting in a calculated activation energy of 0.601 eV. The formation of vacancies in metals also is of the form $N_{\text{vacancies}} = N_0 \exp(-E_a/k_B T)$ and this suggests that vacancies may play a significant role in the performance of a LEC. The activation energy of vacancies in Pd is between 1.41 and 1.52 eV [20].

Figure 12 is included as an example of a self-initiating and self-sustaining LEC cell that changed polarity three times over a 4-day period. The LEC voltage was initially negative and it became positive as the temperature was increased. When the temperature decreased, the LEC voltage went back negative and then it gradually increased to positive even though the temperature remained relatively constant. Load tests conducted when the voltage was negative and when it was positive are similar to load tests for cells that maintain a relatively steady voltage. Similar behavior has been observed in multiple LEC cells. The cause is not known although one possibility might be a change in work function of the materials as hydrogen loading changes and as the electrode surfaces are subjected to ionizing radiation. This behavior is another example of the complex nature of what appears to be a simple device.

4. Analysis of experimental results

4.1. Theory

Although the physical processes that give rise to the ionizing radiation in a LEC are not understood, there is a well-developed theory for the conduction of electricity through gases. In his lectures on *The Discharge of Electricity Through Gases* [9] Thomson writes:

We shall find that the analogy between a dilute solution of an electrolyte and gas exposed to Röntgen rays holds through a wide range of phenomena, and we have found it of great use in explaining many of the characteristic properties of the conduction through gases.

Although their electrical instrumentation was limited by today's standards, and the analysis of the conduction of electricity did not initially include the effects of ion diffusion or space-charge in the gas, Thomson and Rutherford's [8] experiments demonstrated:

The fact that the passage of a current of electricity through a gas destroys its conductivity explains a very characteristic property of the leakage of electricity through gases exposed to Röntgen rays; that is, for a given intensity of radiation the current through the gas does not exceed a certain maximum value whatever the electromotive force may be. the gas gets, as it were, 'saturated.'

A few years later, Thomson [10] gave a more mathematically detailed analysis of the conduction including the effect of space-charge in the gas but still without considering diffusion.

The electrical conductivity possessed by gases under certain circumstances—as for example when Röntgen or uranium rays pass through the gas, or when the gas is in a vacuum-tube or in the neighbourhood of a piece of metal . . . illuminated by ultra-violet light—can be regarded as due to the presence in the gas of charged ions, the motion of these ions in the electric field constituting the current.

Unfortunately, Thomson was not able to integrate the differential equation for the electric field as a function of position within the gas for the general case of a finite rate of ionization, q , and where the positive and negative ions have different mobilities, *i.e.*, have different drift velocities under the influence of the electric field. Thomson gives an approximate method for finding the electric field in the gas when the number of positive ions is equal to the number of negative ions. For the case of the electric field between plane-parallel electrodes in air, he estimates that the electric field at the point between the electrodes where $d^2E/dx^2 = 0$ as $1/2.51$ of the electric field at the same point assuming that there is no space-charge. Also, he gives a formula for finding this ratio for any gas whose mobility values are known and this evaluates to $\sim 1/4.67$ for hydrogen. A few years later, E. Riecke [21] and G. Mie [22] found approximate solutions to the electric field distribution in the gas for various ratios of current density to saturation current density.

In the latter part of the 19th century and the beginning of the 20th century the emphasis of researchers was to understand the physics of conduction of electricity through gases. To this end most researchers took precautions to ensure that their Röntgen (X-) rays or emissions from uranium salts did not fall on the electrodes of their experiments which would produce additional ions due to the photoelectric effect.

With Townsend [15], a research student of Thomson, the emphasis shifted from experiments at low values of electric field to pressure, *e.g.*, $E/p < 2$ where E has units of $[V \cdot cm^{-1}]$ and p is measured in $[mm \text{ Hg}]$ or $[Torr]$, to experiments at high $E/p > 2$ where the influence of the impact of gas ions on the electrodes now became important [23]. Since the discharges were self-sustaining at high $E/p > 2$, the role of external ionizing radiation became less important. Other material that may be useful is Thomson's Encyclopædia Britannica article [24] that includes an analysis of the conduction when all of the ions are of the same sign and thus carry the same charge. Also, to understand some of the electrical engineering issues that are important at high E/p it is instructive to review Peek's [25] '*Dielectric Phenomena in High Voltage Engineering*.' In this book there are discussions of corona discharge and of gas breakdown or arcing in cylindrical coordinates when E/p is high. Since a LEC operates at $E/p < 2$, these phenomena as possible explanations of LEC conduction can be eliminated and the more difficult question of what is causing the conduction can be undertaken.

It wasn't until the work of the research physicist KK Darrow [16], while working at the Bell Telephone Laboratories, that the emphasis on the analysis of the conduction of electricity through gases changed and he wrote:

It is the condition of the conductive gas which matters first and most; we have to analyze it, to identify the various kinds of particles charged and chargeless which make it up, determine their numbers and their speeds, and then (if possible) discover how the applied potential-difference and the other agencies of the environment

bring this state into being. As for the current, that may well be regarded as a minor perturbation of the conducting system.

In his analysis of the conducting system of three equations, following Riecke [21], Darrow includes both the effect of the electric field and of the diffusion of the ions in the gas. He finds that diffusion of ions due to positive and negative ion concentration gradients, $-D_1 dn_1/dx$ and $-D_2 dn_2/dx$ respectively, has the same effect as the drift of ions due to the electric field. Thus, when he writes the equation for the current density per unit area, i , one term involves the product of the density of ions, n , times the electric field, E , and the ions mobility, μ , while the other term involves the gradient of ion density, dn/dx , times the diffusion-coefficient, D , which is by Einstein's relationship for gases, the ion mobility times Boltzmann's constant times the ion temperature in Kelvin, *i.e.*, $D = \mu k_{eV} T$ cm²/s where $k_{eV} = k_B/e[eV \cdot K^{-1}]$ may be substituted in place of k_B so that the conduction equations now become:

$$q - \alpha n_1 n_2 + D_1 d^2 n_1 / dx^2 - \mu_1 d(E n_1) / dx = 0 \quad (1)$$

$$q - \alpha n_1 n_2 + D_2 d^2 n_2 / dx^2 + \mu_2 d(E n_2) / dx = 0 \quad (2)$$

Where q is the ionization rate, n is the number of ion-pairs ionized per second per cubic centimeter, and α the recombination rate which is approximately 1.4×10^{-6} cm³/s for H₂ and 1.6×10^{-6} cm³/s for air.

$$i = e[(n_1 \mu_1 + n_2 \mu_2)E + (D_2 dn_2/dx - D_1 dn_1/dx)] \quad (3)$$

Darrow in chapter V on the "Elementary Theory of Drift" further notes in equations 66, on page 193, reproduced here as (4), that if $n_1 \approx n_2$ then:

by making n_1 and n_2 equal and denoting each of them by n , we get:

$$i/e = n(\mu_1 + \mu_2)E + (D_2 - D_1) dn/dx \quad (4)$$

an equation easy to integrate when the mobilities and the diffusion coefficients are assumed independent of x .

What is peculiarly interesting is that E and i do not necessarily vanish together [emphasis added].

Darrow's equation (4) now explains the surprising inferences that can be drawn from LEC experimental results as shown in Fig. 5 and Fig. 6. In the case of Fig. 5 the current went to zero as the impressed test voltage was approaching zero because the LEC cell was producing a current flowing in the opposite direction to the impressed current as a result of diffusion of the ions due to an ion concentration gradient within the cell. In the case of Fig. 6, *i.e.*, the extrapolation of the current to zero, the cell current effectively goes to zero when a high value of resistance such as a DVM is connected to the cell.

Darrow comments that this result can be interpreted as follows: [emphasis added]

Suppose the ions of the two signs are spread identically through a gas between two walls—by 'identically' I mean that everywhere the concentrations of the two kinds are equal, though their common value varies from place to place—and that this state of affairs is stationary. **Then there must be ionizing rays acting continually on the gas, and also there must be a potential-difference between the walls.** For if there were no field, the negatives would diffuse along the concentration-gradient more rapidly than the positives; there would be a net current; this would result in a depletion of negatives, and an excess of positive charge would arise in the gas, in contradiction with the assumption. But suppose there is a P.D. [Potential Difference] between the walls, in such a sense as to oppose the negatives and pull the positives forward as they stream together down the gradient. A value for this potential-difference can be found, such that the field strength will retard the negatives and encourage the positives just sufficiently to annul the net current aforesaid;

This potential-difference is just the voltage that the DVM measures and thus the ‘peculiar’ behavior of a LEC is explained, including the increase in LEC voltage with temperature since, by the Einstein relationship for gases, the diffusion coefficient, $D = \mu k_B T$, increases with temperature. There could also be a contribution to potential difference due to different work functions of the electrodes but this is not required as Darrow has shown. In summary: the gas in a LEC must contain ions because it conducts electricity; there are approximately an equal number of positive and negative ions since the cell’s conduction remains essentially the same when the polarity of the applied potential reverses; there must be an ion concentration gradient since the cell generates a current when the electric field, E , tends to zero as the load resistance R_L tends to zero.

Thus, Darrow’s inclusion of the drift term in the conduction equation explains the ‘peculiar’ I-V characteristics that were experimentally observed and also indicates that diffusion is one of the processes occurring within a LEC. Not explained is the cause of the spontaneous ionizing radiation or why the ion density in the cell appears to be non-uniform. One possible explanation for the later is that the WE emits ionizing radiation that produces energetic photo-electrons at the CE and these in turn ionize the gas in proximity to the CE thus producing an ion concentration gradient.

There are limitations with the above analysis since assuming in (4) that $n_1 = n_2 = n$ implies that there is no space-charge. This is equivalent to assuming that the electric field, E , is given by the relationships $E(x) = V/L$ for plane-parallel geometry or $E(r) = V/[r \ln(b/a)]$ for cylindrical geometry where $a \leq r \leq b$ and where V is the potential-difference between the electrodes, L is their separation for plane-parallel geometry, and a and b are the inner radius and outer radius of the electrodes respectively with cylindrical electrode geometry.

Darrow writes:

Space-charge would play no part, of course, if n_1 and n_2 were equal; but this is a condition which the difference in the mobility of the two kinds of ions makes difficult to approach—one easily sees that if carriers of both signs are generated at an equal rate, and the negatives drift faster than the positives, there will be fewer negatives than positives at any moment between the plates. However it is interesting to simplify the equations in the way that would be permissible were n_1 and n_2 sufficiently nearly the same.

Experience has shown that this type of approximation is useful since the dominant term in the calculation of radiation-induced current, I_{rad} , is the term (i/e) where $i = (I/S_m)$ which in SI or CGSA units is $\sim 6.2415 \times 10^{18} (I/S_m)$ where I_{rad} is the short-circuit cell current, typically about 10^{-6} A and S_m is the cross sectional area near the middle of the conduction path where $d^2 E(x)/dx^2 = 0$. L.B. Loeb [18] includes information learned up to and including WWII and suggests that there is a “. . . rejuvenation of the field of study previously called The Discharge of Electricity in Gases, and now more properly called Gaseous Electronics.” Of particular importance to the understanding of LEC performance was the development of ionization chambers [26]. An ionization chamber such as a Gerdien Condenser [27] that is designed to measure the number of ions in the gas as it flows through the device, measures the saturation current produced by the radiation flux in a fixed volume of gas when exposed to ionizing radiation. Rossi and Staub [26] calculate the densities of positive and negative ions due to ionization for both the case where recombination and diffusion are neglected and when they are included. A particular type of ionization chamber, *i.e.*, a reentrant well-type or 4pi ionization chamber, is similar to a LEC in that the radiation source to be measured is internal to the chamber in much the same way that the WE of a LEC is internal to the device. Thus, the question of the distribution of the electric field within the gas is important, however, modern computers have made the numerical evaluation of the field due to space charge possible. Several papers [28] [29] [30] [31] describe these recent results for plane-parallel, cylindrical, and spherical electrode geometries. While they also neglect the diffusion terms, they are good at predicting the shunt term. However, even this analysis does not predict I-V characteristics of the LEC experimental measurements which indicate that LEC results are driven by the diffusion of the ions within the gas.

4.2. Estimating LEC cell performance

The estimation of a LEC cell's performance depends on whether the measurements are external voltage induced conduction or spontaneous conduction. For voltage induced conduction, start with a measurement of the I-V characteristics of the cell using the instrumentation configuration shown in Fig. 2 for a number of different supply voltages. A typical value of cathode resistance, R_{cat} , is 10 k Ω and a typical value of the supply voltage current limiting resistance, R_{sup} is 1 M Ω . This ratio of resistances ensures that the maximum voltage presented to the instrumentation system is limited to 10 volts and the maximum current is limited to 1 microampere. The current through the cell is I_{cell} and the voltage across the cell, V_{cell} , is $V_{cell} = V_{anode} - V_{cathode}$. The cell may be installed with either the working electrode (WE) or the counter electrode (CE) as the anode.

An initial estimate of the cell's performance may be made by assuming plane-parallel electrode geometry with separation L and no space-charge in the gas. Then, the electric field is $E = V_{cell}/L$ and the current density is $i = I_{cell}/S$ where S is the cross-sectional area of the conduction path between the electrodes. Let the density of positive ions, n_1 , and negative ions, n_2 , each be equal to $n(x)$ a function of position where x is measured from the anode. This assumption is equivalent to assuming the ionized gas is an ion-ion plasma. Let $U = (\mu_1 + \mu_2)$ where μ_1 is the mobility of the positive ions and μ_2 is the mobility of the negative ions with $\mu_2 > \mu_1$. Let $U_{21} = (\mu_2 - \mu_1)$ and denote Boltzmann's constant as $k_{eV} \approx 8.6173 \times 10^{-5} eV \cdot K^{-1}$, so that the difference in diffusion coefficients is $D_2 - D_1 = U_{21} k_{eV} T$. Then equation (4) may be rewritten as

$$i/e = Un(x)E + U_{21}k_{eV}Tn'(x) \quad (5)$$

with solution

$$n(x) = (i/e)/UE + C \exp(-UEx/U_{21}k_{eV}T) \quad (6)$$

where C is a constant of integration, T is the cell's temperature in kelvin, $1/e \approx 6.2415 \times 10^{18} A^{-1} \cdot s^{-1}$ is the reciprocal of the elementary charge, $e := 1.602176634 A \cdot s$, and $dn(x)/dx = n'(x)$.

An estimate of the magnitude of the ratio of ions at the cathode to ions at the anode can be found by solving eq (5) when $i \approx 0$. After integration $\ln(n_L/n_0) = -Un(L)V|_{I \approx 0}/U_{21}$ so that when $V|_{I \approx 0} > 0$ as in Fig. 5, then $\ln(n_L/n_0) < 0$ and $0 < n_L/n_0 < 1$ and the maximum ion density occurs at the anode which is the CE for the Pd-D cell of Fig. 2. Additionally, let $x = 0$ then the constant C is found to be $C = n(0)|_{I \approx 0} = n_0$.

For the case of spontaneous LEC conduction, only the LEC voltage, $V_{LEC}(R, T)$, is measured as a function of effective load resistance, $R = R_{Load}R_{DVM}/(R_{Load} + R_{DVM})$ and I_{Load} is calculated using Ohm's Law as $I_{Load} = V_{LEC}/R$. From the phenomenological description of a LEC in Fig. 11, $I_{Load} = I_{Radiation} - I_{Shunt}$. Since i_{Shunt} is the internal current through the gas caused by the drift of the ions due to the electric field, $E(x)$, so that i_{Shunt} may be estimated as $neUE(x)$. A general analytic expression for the electric field is not known. However, as is shown by Thomson [10], [12], and Thomson and Thomson [13], $n_1 \approx n_2 \approx n$ is correct only when $dE/dx \approx 0$ which occurs away from the electrodes. In a cylindrical electrode geometry LEC with close electrode spacing, the approximation $dE/dx \approx d(rE)/dr = 0$ may not be valid. Thus $E(x) \neq V_{LEC}/L$ and $i_{Radiation}$ must be estimated first since $i_{Shunt} = i_{Radiation} - i_{Load}$.

Since the magnitude of drift velocity of the ions is $|v_1(x)| = \mu_1 E(x)$ and $|v_2(x)| = \mu_2 E(x)$, as R is reduced, V_{LEC} and $E(x)$ are also reduced which results in a decrease in drift velocity. This in turn leads to an increase in the time that the positive and negative ions have to recombine and no longer contribute to the cell's conduction. This is shown in Fig. 9 as a reduction of the measured $I_{Radiation}(R, T)$ for small R . Observe that $I(R, T)$ may vary for small R so select its maximum value I_{max} . Assume that there is no space charge, i.e., from Gauss' law, $\text{div } E(x) = \rho/\epsilon = e(n_1 - n_2)/\epsilon = 0$, where E is the electric field, ϵ is the permittivity of the gas and ρ is the volume charge density or charge per unit volume. Rewrite the left-hand side of equation (4) in terms of $i_{Radiation}$ as

$$(i_{Radiation}/e) = (I_{max} + \epsilon)/Se = n_{Average} \quad (7)$$

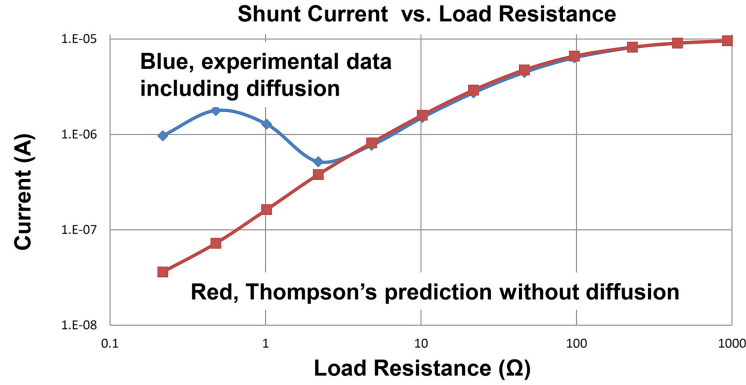


Figure 13. Plot of the predicted shunt current by Thompson (Red) which does not include the diffusion terms and experimental data which includes diffusion.

where in CGSA units $6.2415 \times 10^{18} \approx 1/e$, $e := 1.602176634 \times 10^{-19} \text{ A} \cdot \text{s}$ is the elementary charge, $I_{\max} [\text{A}]$ is the measured LEC cell's load current when $V_{\text{LEC}}[\text{V}] \approx 0$, epsilon is a small number, $S [\text{cm}^2]$ is the cross-sectional area of the conduction path, $n [\text{ip} \cdot \text{cm}^{-3}]$ is the average ion-pair density $n_{\text{Average}} = \int_0^L n(x) dx / L$. The geometry is assumed to be plane-parallel electrodes since this last assumption can be corrected by a change of variables [32] and usually is a small error even if not corrected.

The cell's internal shunt current density, $i_{\text{shunt}}(R)$ may be found by subtracting $i_{\text{load}}(R)$ from $i_{\text{rad}} = I_{\max}/S$ after adding a small number, epsilon, to i_{rad} in order to be able to make a semilog plot of $i_{\text{shunt}}(R)$ versus R without encountering a zero value of $i_{\text{shunt}}(R)$. An estimate of the error made in assuming that there is no space charge can be made by calculating a hypothetical $i_{\text{hypot}}(R) = n_{\text{hypot}} \cdot U \cdot V_{\text{LEC}}(R)$ and varying n_{hypot} until it overlays $i_{\text{shunt}}(R)$. This is shown in Fig. 13. That the ion density $n \neq n_{\text{hypot}}$ is different in the two calculation is the result of Thomson's [12] and Mie's [22] finding that when there is space charge $E \neq V/L$.

The LEC's power, $P(R, T)$, delivered to the effective load resistance, $R = R_{\text{Load}} R_{\text{DVM}} / (R_{\text{Load}} + R_{\text{DVM}})$, at temperature T is just $P(R, T) = V_{\text{LEC}}^2 / R$. Figure 10 experimentally illustrates this behavior, and the value of maximum power occurs at an intermediate value of load resistance.

5. LEC performance observations

Although both the origin as well as the nature of the LEC radiation that ionizes the gas and provides the measured spontaneous voltage and current is unknown, certain observations about the performance of a LEC can be made. Some observations about LEC performance include:

- (a) a necessary but not sufficient condition for sustained LEC operation is that the working electrode must be comprised in part of hydrogen-occluded hydrogen host material that is in fluidic contact with a gas containing hydrogen;
- (b) hydrogen host material lattice vacancies, superabundant vacancies, and other defects such as those produced during co-deposition of Pd or Fe from an aqueous solution have been shown to produce ionizing radiation from an active working electrode;
- (c) a necessary condition is that the gas be comprised in part of hydrogen, or its non-radioactive isotope deuterium, although sustained conduction at a low level has been observed in air indicating that the amount of hydrogen may not need to be large;

(d) reconnecting the electrodes from that shown in Figure 2 to reverse the polarity of the electrical potential, *e.g.* high voltage supply connected to the working electrode and load resistance connected to the counter electrode, causes a temporary increase in conductivity but does not appreciably change the long term steady state conductivity;

(e) voltage induced conduction experiments using deuterium gas at $T \approx -55^\circ\text{C}$ show that ions in the gas due to relative humidity are not causing the conduction since at this temperature the number of water (D_2O) ions is completely negligible;

(f) corona discharge cannot be responsible for the conduction since both the electric field due to the spontaneous voltage is too low to produce corona discharge [Peek 1929] and the gas pressures, typically ~ 500 Torr to ~ 3 bar is too great for a discharge;

(g) thermal ionization of the gas is not responsible for the LEC conduction since both reported data [34] and Saha's equation [35] shows that temperatures greater than approximately 2000 K would be needed;

(h) natural ionizing radiation from cosmic rays, the environment, or from the small amount of radioactive isotopes in the hydrogen host material or from tritium in the hydrogen gas cannot be the source of the ionization of the gas since an experiment where the working electrode was bare but contained approximately $6\ \mu\text{Ci}$ ($\sim 90\%$ α and $\sim 10\%$ γ) of radiation did not produce measurable conduction from the ions produced by the α -particles or from the photoelectric effect due to the γ -radiation produced by the WE;

(i) after extensive testing, Rout, *et al.* [36] were unable to identify the specific ionization that was fogging the film and ultimately, "proposed that some new, unknown agency emitted from the loaded palladium is responsible for fogging."

Possible physical mechanisms that might produce the initial ionizing radiation at the working electrode include:

(a) thermally induced vibration of the hydrogen host material's lattice enhanced by nonlinear wave-wave mixing in conjunction with the presence of hydrogen occluded in lattice vacancies near the surface of the hydrogen host material;

(b) thermally induced interaction between multiple occluded hydrogen atoms contained within a vacancy particularly when different nuclear spin orientations are present or between an occluded hydrogen atom in the vacancy and a hydrogen host material's atom.

6. Conclusions

A number of LEC devices have been constructed including independent replications which produce spontaneous voltage and current. Although the measurements only consist of the voltage as a function of time and temperature produced across various load resistances the following conclusions can be drawn from an extended mathematical analysis of the measurements:

(a) the spontaneous current produced by a LEC is essentially constant and is primarily due to the diffusion of ions. This diffusion induced current can be measured when the LEC voltage is minimized by resistance loading of the cell and the corresponding ion concentration can be estimated using an induced conduction test and measuring the voltage at which the induced current goes to zero as shown in Fig. 5;

(b) From the theory of the *Conduction of Electricity through Gases* it is known that in the absence of emitted particles, *e.g.*, radioactive decay particles, thermal electrons, or photo-electric effect electrons, the gas must be ionized and the average ionization density can be calculated. This historical theory may also be used to estimate the shunt current in the cell due to the ion mobility and the electric field and can be estimated, as a function of temperature, based on the measured current-voltage (I-V) characteristics as shown in Fig 9;

(c) for a Pd-H LEC at 185°C ion average densities greater than 10^{10} ion-pairs (ip) per cubic centimeter have been calculated from experimental measurements;

(d) at a fixed temperature the magnitude of the spontaneous current flowing in an attached load resistance decreases at the same time that the shunt current increases due to a voltage increase with increasing load resistance value. Thus, the maximum power that can be drawn from a LEC occurs at some intermediate resistance value as shown in Fig 10.

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