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Research Article

Atomic Nuclei Binding Energy: Case of $_{26}$Fe Isotopes

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Abstract

In 1936. Bethe and Bacher and in 1938 Hafstad and Teller predicted that αparticle structures could be present in atomic nuclei. In the course of developing a theory of nuclear structure based on the assumption that clusters of nucleons are packed as closely together as possible, Linus Pauling found that the magic numbers have a very simple structural significance. He assumed that in nuclei the nucleons may, as a first approximation, be described as occupying localized 1s orbitals to form small clusters. These small clusters, called spherons, are usually helions (i.e. α particles), tritons and dineutrons. In nuclei containing an odd number of neutrons, a ^{3}He cluster or a deuteron may serve as a spheron. The close-packed-spheron model differs from the conventional liquid-drop model of the nucleus in having spherons rather than nucleons as the units. This is a simplification: ^{154}Gd, for example, is described in terms of 45 spherons, rather than 154 nucleons. This enables one to determine the systematic of binding energy in a much simpler way than the approach based on individual nucleons. The author developed that idea, i.e. having clusters as basic bricks within the nucleus instead of nucleons. He considered the binding energy of α particle, deuterium, tritium, ^{3}He, and the way these spherons are bonded instead of the bonding between individual nucleons. According to that hypothesis the nuclei of the various isotopes of each element are constituted out of α particles and other nucleons grouped in order to form sub-nuclei bound together by four types of bonds called NN, NP, NNP and NPP. So, the author favored an approach trying to breakdown the binding energy value of each element isotope in the sub-values indicated above. That binding energy distribution approach in the nuclei is essential to the comprehension of LENR process.
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1. Introduction

Several authors [1] predict that αparticle structures could be present in atomic nuclei. Convincing arguments of such structures are provided by systematics of the binding energies of the even–even nuclei with equal number of protons and neutrons. So, a first point to consider concerns the binding energy (E_{B}) of an α particle and its relationship with the binding energies of deuterium, tritium, and helium-3, which are nuclear clusters smaller than helium-4. A second point is to see if and how these three binding energies play a role in the bonds between the α particles, binding a

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nucleon of one α particle to a nucleon of a second α particle. These issues have already been treated in articles by the same author [2]

The author tried to organize the nucleus in a way similar to Pauling's model of the nuclear structure, with some clusters within the nucleus Pauling called spherons. The sub-nuclei taken into consideration are the α particles and four types of bonds, determined in the following way:

- Deuterium-like bond, called NP with value 2.2246 MeV, linking a neutron of one α particle with a proton of a second α particle, or a neutron or proton outside an α particle to that α particle.
- Tritium-like bond, called NNP with value 8.4818 MeV, linking three nucleons of three different α particles, or one or two nucleons outside an α particle to one or two α particles.
- ^{3}He-like bond, called NPP with value 7.718 MeV, having a similar function as NNP.
- A dineutron bond, called NN, with value 4.9365 MeV and linking two neutrons not being located within the same α particle. This bond and its value are deduced from the α particle binding energy.

So, the binding energy (E_{B}) of an isotope is composed of the E_{B} of the α particles (28.325 MeV each) together with the E_{B}'s of the various four bonds determined above.

The case of the light element's isotopes like ^{16}O, ^{20}Ne, ^{24}Mg, ^{28}Si, ^{32}S, ^{36}Ar, and ^{40}Ca was already treated [2]. There are by definition only α particles within these nuclei. The problem to solve is how they are bound together. For instance, ^{8}Be is not stable, as there is no room for bonds between the two α particles, the E_{B} of that isotope being more or less equal to the E_{B} of its two α particles. It is not the case of ^{16}O containing four α particles and having a global E_{B} superior to the E_{B} of these four α particles together. This difference represents the E_{B} between the four α particles.

Important remark: The binding energy taken into consideration in this paper is the net binding energy, i.e. the Coulomb energy once deducted from the total strong interaction. The Coulomb energy is a repulsive energy, whereas the nuclear binding energy is an attractive energy. Thus, the actual total energy required to bind nucleons into a nucleus and to overcome the repulsive Coulomb energy is the sum of these two. In other words, the binding energy is the net energy that is left over after the repulsive Coulomb energy is overcome. So, the binding energy taken into consideration in this paper is the energetical counterpart of the mass defect, the first being expressed in MeV, the second in atomic mass units.

1.1. Composition of inter alpha binding energy

The author assumes that the bonds between α particles should link two α in case of NN and NP involving only two nucleons, and three α in case of NNP and NPP bonds involving three nucleons. So, the bonds of type NNN and PPP types are eliminated as not "realistic" and NP, NNP, and NPP are accepted because they are equivalent to deuterium, tritium and ^{3}He bonds already existing before the α particle is constituted.

As far as NN and PP are concerned, these being part of the α particle binding energy, only NN is accepted as an inter alpha bond for the following reasons.

- With the exception of ^{3}He there are no stable nuclides containing more protons than neutrons. So, for the stable nuclides, there is only one possible proton outside the α particles. There could, of course, be more neutrons. This fact excludes proton–proton bonds outside an α particle.
- Coming back to ^{16}O and to the binding energy in excess to that of the α, the following is noticed: The four α's could be linked by a minimum of three bonds, one bond between each α. Or similarly, by one NNP or NPP bond together with one NN or one NP bond. However, the only suitable values are two NP bonds,

together with the neutron–neutron (2 NN) binding energy within α particle. The binding energies of four alpha particles, plus two NP binding energies, plus the "neutronic" (2 NN) part of the α particle binding energy can be summed together, and this sum is equal to the binding energy of ^{16}O. The NN value is deduced from the neutron-neutron binding energy within α particle.

These are the assumptions concerning the four bonds NN, NP, NNP, and NPP. To simplify the author merges NP and NN in one bond called A (for average) = (NN/2+NP/2).

The key idea of the theory is to find a common distribution of binding energy within the various isotopes which could in turn help to understand the LENR process. It is about finding a kinship between these various nuclides. According to the theory there is the following sequence in binding energy:

NP ⎫ bonds smaller than
NNP ⇒ NPP ⎭ α particle bond
α particle bond

So in the case of two or more α particles in a nucleus, it is assumed that the binding energy between these α particles is based on bonds linking their nucleons, and that the value of these bonds is related to the values of NP, NNP, NPP and NN bonds, all of which are part of the α particle binding energy.

1.2. Progression of binding energy

1.2.1. *Basic values*

NP $= 2.2246$ MeV,

NNP $= 8.4818$ MeV,

NPP $= 7.7180$ MeV. Difference between NNP and NPP = 0.7638 MeV,

E_{B} Alpha particle $= 28.325$ MeV,

NN $= 4.9365$ MeV.

1.2.2. *Determination of binding energy values based on preceding values*

^{5}He $=^{4}$He+ N (neutron),

$E_{\mathrm{B}} = E_{\mathrm{B}}\alpha -$ NNP + NPP $= 28.325 - 0.7638 = 27.5612$ MeV,

^{6}Li $=^{4}$He+ N + P (proton),

$E_{\mathrm{B}} = E_{\mathrm{B}}\,\alpha -$ NNP + NPP + 2NP $= 28.325 - 0.7638 + 4.4492 = 32.01$ MeV,

^{7}Li $=^{4}$He+ 2N + P,

$E_{\mathrm{B}} = E_{\mathrm{B}}\alpha +$ 3NP + NNP/2 $= 28.325 + 6.6738 + 4.2409 = 39.2397$ MeV,

^{9}Be $= 2\ ^{4}$He+ N,

$E_{\mathrm{B}} = 2\ E_{\mathrm{B}}\,\alpha +$ 1.5/2 NN + 1.5/2 NP − NPP/2 $= 56.65 + 3.7024 + 1.6685 - 3.859 = 58.162$ MeV,

^{10}B $= 2\ ^{4}$He+ N + P,

$E_{\mathrm{B}} = 2\ E_{\mathrm{B}} \quad \alpha +$ NNP/2 + NPP/2 = 56.65 + 4.2409 + 3.859 = 64.7499 MeV,

^{11}B = 2 ^{4}He+ 2N + P,

$E_{\mathrm{B}} = 2\ E_{\mathrm{B}} \alpha +$ 1.5 NN +2 NP + NPP = 76.222 MeV.

1.2.3. *Other examples of nuclei binding energy*

E_{B} ^{12}C =3 E_{B} $\alpha +$ NN + NP = 92.136 MeV,

E_{B} ^{13}C =3 E_{B} $\alpha +$ NN + NP + NP/2 + NPP/2 = 97.107 MeV,

E_{B} ^{14}C =3 E_{B} $\alpha +$ 1.5 NP + 2NNP = 105.274 MeV,

E_{B} ^{15}C =3 E_{B} $\alpha +$ NP + 2.5 NPP = 106.495 MeV,

E_{B} ^{16}C =3 E_{B} $\alpha +$ NP + 2.5 NPP + NNP/2 = 110.735 MeV,

E_{B} ^{14}N =3 E_{B} $\alpha +$ NNP/2 + 2 NPP = 104.652 MeV,

E_{B} ^{15}N =3 E_{B} $\alpha +$ 2NN + 2NP + NNP + NPP = 115.497 MeV,

E_{B} ^{16}N =3 E_{B} $\alpha +$ 1.5NN + 2.5NP + NNP + 1.5NPP = 118.000 MeV,

E_{B} ^{16}O =4 E_{B} $\alpha +$ 2NN + 2NP = 127.622 MeV,

E_{B} ^{17}O =4 E_{B} $\alpha +$ 1.5NN + 1.5NP + NPP = 131.760 MeV,

E_{B} ^{18}O =4 E_{B} $\alpha +$ 2NN + 4NP + NPP = 139.789 MeV.

Remark: for all these results the differences between experimental and calculated values are less than 0.026 MeV (source: "The AME 2016 atomic mass evaluation") [3].

1.2.4. *Determining the binding energy of any isotope*

These results are obtained by comparing binding energy values of isotopes of the same element and by breaking down these values in NP, NNP, NPP, and α particle binding energy values. The α particle binding energy value is also broken down in 2 NN and PP values. Only NN is active outside the α particle, PP being active within α particle. One single process is used by the author, i.e. looking step by step, isotope of one element after isotope of the same element, for binding energy differences. One should also consider that the mass differences in binding energy values could be positive or negative, the negative values showing a mass reconstitution. Having this in mind one can determine the binding energy value of every nuclide. See the figures displayed in www.philippehatt.com [4]. The geometrical schemas displayed on the author's website are not designed to build a structure of nuclei, but rather are designed to be a visual support for research, especially to see the kinship between the binding energy distribution among the various nuclides. For instance, in case of ^{16}O the figure is based on four α particles bound by four equal bonds called "A", actually the average of NN and NP (NN/2 + NP/2). If a neutron is added it becomes ^{17}O. The author looks for a bond connecting the new neutron to two nucleons located within two of the α particles in the ^{16}O structure. This is the state closest to ^{16}O. The choice is between NNP and NPP. It is NPP which fits, so that bond is taken arbitrarily. Actually, the author uses the three bonds corresponding to the three nuclides pre-existing to the α particle, i.e. NP, NNP, NPP, and a fourth one deduced from α particle bond, i.e. NN.

So, this method is not based on a theory. Instead, it is based on mind experiments, and choosing between a few bonds each time a new neutron or proton enters a nucleus. You choose that one which fits. This unconventional way

is comparable to the work of a chemist looking for several solutions in his experiments and validating that one which fits best. .

Moreover, the author is looking at the compliance of the solution for one nucleus with the solution for another nucleus in order to avoid discrepancies, especially between isotopes of the same element. In doing so, the important thing is symmetry within isotopes of one element and hence between all element's isotopes. Indeed, this work does not address the three-dimensional model of nuclei in the sense that the author is not looking for a structure of these nuclei but rather for the distribution of binding energy among them. Nevertheless, this work could be complementary to those dealing with that topic. The author's work is trying to explain the LENR processes where energy release is a direct consequence of nuclear transmutations, i.e. modification of binding energy values between isotopes present at the beginning and at the end of the LENR reaction.

1.2.5. *Calculation of binding energy*

As seen, the system for determining the binding energy of the nuclides is based only on calculations and similarities between isotopes of one element or between isotopes of elements close from each other.

Two examples of calculations: E_{B} of ^{8}Be and the reason for its instability. E_{B} of ^{9}Be and the reason for its stability:

^{8}Be	$2 \times 28.325 =$	56.6500	$2\times 28.325 =$	56.6500
	NN/2 =	2.4683	$-$NNP/2 =	–3.8590
	NP/2 =	1.1123		
		60.2306 MeV		52.7910 MeV

Average: 60.2306/2 + 52.7910/2 = 56.511 MeV (+0.011 MeV compared with AME 2016 value).

Explanation: the balance between (NN/2 + NP/2) and NNP/2 is negative (2.4683 + 1.1123 - 3.8590 = −0.2784 MeV). Therefore, there is no bonding possibility between the two α particles and the nucleus is unstable as the reconstitution of mass is more important than the mass defect.

^{9}Be	$2 \times 28.325 =$	56.6500	$2 \times 28.325 =$	56.6500
	3 NN/2 =	7.4048	$-$NPP =	–7.7180
	3 NP/2 =	3.3369		
		67.3917 MeV		48.9320 MeV

Average: 67.3917/2 + 48.9320/2 = 58.1640 (+0.002 MeV compared with AME 2016 value).

Explanation: the introduction of one neutron has occurred in two more A i.e. (NN/2 + NP/2) bonds and also a negative NPP bond double as for ^{8}Be. Nevertheless, the balance is positive (7.4048 + 3.3369 − 7.7180 = 3.0237 Mev). The nucleus is stable.

1.2.6. *Value of a "line"*

The author looks for a relationship between the four values: NN, NP, NNP, and NPP in order to determine a common basic value unit of binding energy. As a result, the binding energy of each nuclide is based on the number of α particles binding energy, multiple of 28.325 MeV, and on a number of "lines" to be broken down in NN, NP, NNP, and NPP.

The following is noticed:

E_{B}	α =	2 NN	9.873
		NP	2.224
		NNP	7.720
		NPP	8.482
			28.299 MeV

E_{B} (NN + NP) =	4.9365	
	2.2246	
	7.1611 MeV	

E_{B} NPP = 7.718 MeV,

Difference E_{B} NPP - E_{B} (NN + NP) = 0.557 MeV,

E_{B} NP = 2.2246 MeV = 4 × 0.556 MeV,

So, E_{B} NPP = E_{B} (NN + NP) + 0.25 EB NP,

As well, E_{B} NN/ E_{B} NP = 4.9365/2.2246 = 2.219. 2.219 × 4 = 8.875 "lines". 8.875 × 0.556 MeV = 4.9365 MeV = E_{B} NN,

E_{B} NNP = 8.4818 MeV. 8.4818/0.556 = 15.25 "lines",

One line is equal to 0.556 MeV; it is the basic E_{B} value, which allows us to calculate the E_{B} of any nuclide.

1.2.7. *Simplification of calculations*

So, to determine the binding energy for each nucleus one needs:

- To use two E_{B} values: the α particles one (28.325 MeV) and the four others (NN, NP, NNP, and NPP) taken all together.
- To look between NN, NP, NNP, and NPP which one suits. In first place the A bond (NN/2 + NP/2) is used. If there is a rest of lines, it will be distributed between NNP and NPP.

Examples

E_{B} ^{16}O = 4 E_{B} α+ 25.75 lines = 4 E_{B} α+ 2 NN + 2 NP = 127.6222 MeV (+ 0.003) [3],

E_{B} ^{35}Cl = 8 E_{B} α+ 128.75 lines = 8 E_{B} α+ 10 NN + 10 NP = 298.211 MeV (+ 0.001) [3],

E_{B} ^{63}Cu = 14 E_{B} α+ 278.375 lines = 14 E_{B} α+ 13 NN + 13 NP + 8 NPP = 551.3883 MeV (+ 0.004) [3].

So, according to the author's theory, the nuclei of the various elements are constituted with α particles and other nucleons grouped in order to create sub-nuclei linked by four types of bonds called NN, NP, NNP, and NPP.

For more details, refer to author's former articles [2] where it is shown that the hypothesis of α structures in the α cluster nuclei can, indeed, describe the binding energy systematics. In such an approach, the system in its ground state behaves like a crystal, with stationary configuration and shape and with defined bond values between the various α particles and/or sub-nuclei.

According to author's hypothesis the binding energy of every nuclide is the sum of the binding energy of its different sub-structures and the binding energy among these sub-structures.

This method of determining the binding energy is totally different from those used up to now.

1.3. Values used to calculate binding energy

$E_{\mathrm{B}}\alpha$	=	28.325 Mev		
E_{B} NN	=	4.9365 MeV	= 8.875 lines	
E_{B} NP ($^{2}_{1}$H)	=	2.2246 MeV	= 4 lines	One line =0.5561589 MeV [2]
E_{B} NNP ($^{3}_{1}$H)	=	8.4818 MeV	= 15.25 lines	
E_{B} NPP ($^{3}_{2}$H)	=	7.7180 MeV	= 13.875 lines	

The NN and NP are the most used bonds. Generally, they oscillate to form an "A" (A for average) bond equal to (NN/2 + NP/2). The NN value (4.9365 MeV) is comparable to the mass of quark down (4.8 MeV) and the NP value (2.2246 Mev) is comparable to the mass of quark up (2.4 MeV).

The NP, NNP and NPP values are those of "The AME 2016 atomic mass evaluation" with four digits after decimal point [3].

1.4. Basic rule for determining the nuclei binding energy

1.4.1. *According to author's theory*

- The calculation of binding energy of the stable isotopes of each light element satisfies the following rule:
  $E_{\mathrm{B}}n$ = xE_{B} $\alpha + x$ E_{B} (NN/2 + NP/2) $+y$ E_{B} (NN + NP)
  (x = number of α particles, y = number of N and P supplementary within a given nucleus).
  This is the case of ^{16}O.
- The basic rule valid for the stable isotopes of elements located at the center of the periodic table, is:
  $E_{\mathrm{B}}n = x$ E_{B} $\alpha + (x + y)$ E_{B} (NN + NP).
  One should note that (NN + NP) bonds could be replaced by NNP or NPP bonds which have a higher energy value.
  This is the case of E_{B} ^{63}Cu.

Some bonds are not completed, (NN/2 + NP/2) being replaced by NP. This happens in case of non-stable isotopes of an element.

- The rule for the heaviest nuclei is close to the first formulation, valid for the light element's isotopes.

Remark: As the number of nucleons increases the equation valid for the light nuclei becomes close to that one for the isotopes of elements in the center of the Table of elements.

^{35}Cl = 8 E_{B} α + 10 NN + 10 NP instead of 8 EB α + 8 NN + 8 NP (+ 2 NN + 2 NP for the two N supplementary linked to the α particles) + NPP.

As seen, the difference is only one NPP. Indeed, this is close to the rule for the isotopes of elements located at the center of the Table of elements. Nevertheless, if E_{B} ^{35}Cl was based on the same rule as for those ones its E_{B} should be 8 E_{B} α + 8 (NN + NP) + 2 (NN + NP) + NPP = 8 E_{B} α + 10 NN + 10 NP + NPP.

One NPP is lacking showing that the cross bonds are not completed.

1.4.2. *The constraints*

For each intermediate-sized nucleus located at near center of the nuclear table, each α particle is linked linearly to another α particle with one A bond. Also, each α particle is linked transversally to another α particle with one A bond. So, Fe which E_{B} is studied in the following has 13 A linear bonds and 13 A cross bonds because it contains 13α particles. Nevertheless, A bonds could be replaced by NP, NN, NNP, and NPP bonds.

The N supplementary are linked to two α particles by (NP + NN) bonds or by NNP or NPP bonds.

The P supplementary is linked by NP or NPP bonds, exceptionally by an NNP bond, to one or two α particles.

1.5. Three remarks

- Geometrical structure of atomic nuclei.
  In author's theory the binding energy of the nuclei has a unidimensional value, broken down by E_{B} α, NN, NP, NNP, and NPP. So, this work does not address the three-dimensional model of nuclei in the sense that it is not looking for a structure of these nuclei but rather for the distribution of binding energy among them. Nevertheless, a comparison with 3D nuclei models is attempted in this article showing the various isotopes of Fe behaving like crystals with a stationary configuration and shape, and with defined bond values between the various α particles and other sub-nuclei.

- It is important to know the distribution of the binding energy in each isotope at the beginning and the end of a LENR fusion/transmutation reaction in order to follow accurately that process.
- Studying all the isotopes of Fe could help knowing what happens in the heart of such a reaction. The same exercise was already made with the same results for the isotopes of elements from Ca up to Ag. The work goes on. A similar work could be undertaken with isobars having the same mass number but different atomic numbers.

Table 1. Binding energy (E_{B}) distribution for ^{52}Fe to ^{61}Fe

Nucleus	Structure	Status	E_{B} Core				E_{B} N, P supplementary				Difference with the basic rule, in lines		
			NN	NP	NNP	NPP	NN	NP	NNP	NPP	Total of lines	Average	Difference
^{52}Fe	13 α	Lifetime: 8.2 h	5	17	2	0					142.8750	167.3750 = 26 A	–24.5000
	0 N, 0 P	Mode of decay: β^+, EC					0	0	0	0	0	0	–
^{53}Fe	13 α	Lifetime: 8.5 min	1	13	3	3					148.2500	167.3750 = 26 A	–19.1250
	1 N, 0P	Mode of decay: β^+, EC					0	0	0	1	13.8750 =NPP	12.8750 =2 A	+1.0000
^{54}Fe	13 α	Stable	12	12	0	1					168.3750	167.3750 = 26 A	+1.0000
	2 N, 0 P	Nat. abundance: 5.8%					2	0	0	0	17.7500= 2 NN	25.7500 = 4 A	–8.0000
^{55}Fe	13 α	Lifetime: 2.6 years	14	12	0	0					172.2500	167.3750 = 26 A	+ 4.8750
	3 N, 0 P	Mode of decay: EC					3	1	0	0	30.6250 = 3 N+1 NP	38.6250 = 6 A	–8.0000
^{55}Fe Fig. 2	13 α	Lifetime: 2.6 years	14	10	0	0					164.2500	167.3750 = 26 A	– 3.1250
	3 N, 0 P	Mode of decay: EC					3	3	0	0	38.6250 = 6 A	38.6250 = 6 A	–
^{55}Mn	12 α	Stable	14	10	0	0					164.2500	154.5000 = 24 A	+9.7500
	6 N, 1 P	Nat. abundance: 100%					0	1	3	3	91.3750 =1NP + 3 NNP +3 NPP	91.1250 =12 A+NPP	+0.2500
^{55}Mn Fig. 2	12 α	Stable	12	12	0	0					154.5000	154.5000 = 24A	–
	6 N, 1 P	Nat. abundance: 100%					0	0.5	6.5	0	101.1250 = 6.5 NNP +0.5 NP	91.1250 =12 A +NPP	+1.0000
^{55}Mn Fig. 3	12 α	Stable	12	12	0	0					154.5000	154.5000 = 24A	–
	6 N, 1 P	Nat. abundance: 100%					0	1	0	7	101.1250 = NP+7NPP	91.1250 = 12 A +NPP	+10.0000
^{55}Mn Fig. 4	12 α	Stable	8	8	0	4					158.5000	154.5000 = 24A	+4.0000
	6 N, 1P	Nat. abundance: 100%					0	0	0	7	97.1250 =7 NPP	91.1250 = 12 A +NPP	+6.0000
^{56}Fe	13 α	Stable	13	13	0	0					167.3750	167.3750 = 26 A	–
	4 N, 0 P	Nat. abundance: 91.8%					3.5	2.5	0.5	0.5	55.6250 = 5 A +NN +NNP/2 + NPP/2	51.500 = 8 A	+4.1250

Nucleus	Structure	Status	E_B Core NN	NP	NNP	NPP	E_B N, P supplementary NN	NP	NNP	NPP	Difference with the basic rule, in lines: Total of lines	Average	Difference
^{56}Fe Fig. 2	13 α	Stable	7	9	0	5					167.5000	167.3750 $=$ 26 A	$+0.1250$
	4 N, 0 P	Nat. abundance: 91.8%					0	0	0	4	55.5000 = 4 NPP	51.5000 =8 A	+4.0000
^{57}Fe	13 α	Stable	13	13	0	0					167.3750	167.3750 $=$ 26A	–
	5 N, 0 P	Nat. abundance: 2.1%					0	0	0	5	69.3750 =5 NPP	64.3750 =10 A	+5.0000
^{58}Fe	13 α	Stable	13	13	0	0					167.3750	167.3750 $=$ 26 A	–
	6 N, 0 P	Nat. abundance: 0.3%					1.5	0.5	2	3	87.4375 = A+NN 2 NNP +3 NPP	77.2500 =12 A	+10.1875
^{59}Fe	13 α	Lifetime: 45.1 days	13	13	0	0					167.3750	167.3750	–
	7 N, 0 P	Mode of decay: β^-					2	2	3	2	99.2500 = 4 A+ 3 NNP +2 NPP	90.1250 = 14 A	+9.1250
^{59}Co	13 α	Stable	13	13	0	0					167.3750	167.3750 $=$ 26A	–
	6N,1 P	Nat. abundance: 100%					3.5	2.5	3	1	100.6875 =5 A + NN +3 NP+NPP	91.1250 =12 A +NPP	+9.5625
^{59}Co Fig. 2	13 α	Stable	12	12	1	0					169.7500	167.3750 $=$ 26A	+ 2.3750
	6N, 1 P	Nat. abundance: 100%					0	1	3	3.5	98.3125 $=$ NP $+$ 3 NNP $+$ 3.5 NPP	91.1250 =12 A +NPP	+7.1875
^{60}Fe	13 α	Lifetime: 3×10^5 years	13	13	0	0					167.3750	167.3750 $=$ 26 A	–
	8 N, 0 P	Mode of decay: β^-					0	0	3	5	115.1250 = 3 NNP + 5 NPP	103.0000 =16 A	+12.1250
^{61}Fe	13 α	Lifetime: 6 min	13	13	0	0					167.3750	167.3750 $=$ 26A	–
	9 N, 0 P	Mode of decay: β^-					2.5	2.5	2	4.5	125.1250 = 5 A +2 NNP + 4.5 NPP	115.8750 = 18 A	+9.2500

2. Binding Energy Distribution among ^{52}Fe to ^{61}Fe

3. Analysis of Table 1

(1) The zone of stability stretches from - 7 lines (^{54}Fe) to + 10.1875 (^{58}Fe)
The most abundant isotope is ^{56}Fe with + 4.125 lines.

(2) Number of N supplementary.
The isotopes which are stable contain 2N, 4N, 5N, and 6N.
^{55}Fe with 3 N supplementary is not stable but has a high lifetime.
^{60}Fe with 8 N supplementary has a very high lifetime.

(3) The presence of N supplementary has as consequence the increase of the value of the binding energy of a given nucleus:

– 24. 5000 lines in case of ^{52}Fe which has no N supplementary.

+10.1875 lines for ^{58}Fe.

+12.1250 lines for ^{60}Fe.

But + 9.25 lines for ^{61}Fe.

(4) As a consequence of the increasing number of N supplementary there is a progressive increase of NN bonds versus NP ones in order to achieve a maximum of parity between both bonds to create as many A bonds as locations (26 in case of $_{26}$Fe).

(5) ^{52}Fe (see Table 1 and Fig. 1 in the appendices)

There are 26 linear and cross-bonds, but only 10A bonds (instead of 26) linking the α particles on the top and the bottom of the structure. Also, 2 NNP are replacing 4A bonds. The cross-bonds are of NP type. Actually, there are some bonds missing in order to achieve the shape of the structure. The table indicates a deficit of 24.5 lines. The structure cannot be stable until the linear and cross-bonds are of A type, hence the EC and β^+ decay.

(6) ^{53}Fe (see Table1 and Fig. 2)

This structure is similar to that one for ^{52}Fe.

Nevertheless, more linear bonds are of NNP and NPP type. There is a deficit of 19.125 lines in the core which shows that the presence of one N supplementary has occurred an increase of the number of lines: + 5.375 in the core of the structure if compared with ^{52}Fe. This structure is not stable for the same reasons as ^{52}Fe.

(7) ^{54}Fe (see Table 1 and Fig. 3)

The core of this structure is almost perfectly shaped, hence its stability. The 2 supplementary N are bound to the core with each 1 NN bond. Even if there is a global deficit of 7 lines, it allows the structure to be stable. The NPP bond on the top and the 2 NN bonds of the 2 N supplementary are interchangeable to assume the stability.

(8) ^{55}Fe (see Table 1 and Figs. 4 and 5)

This structure is comparable to ^{54}Fe. Nevertheless, to achieve the bonding of the 3 N supplementary it is

necessary to use at minimum 6A bonds (3 × 2A). The 2 NN bonds on the top of the structure and the 2 NN bonds of the 2 N supplementary are interchangeable. Nevertheless, if one wants to interchange the 3 N supplementary bonds with the structure of the top, the result is 6A bonds for the 3 N supplementary and 4 NN bonds for the top of the structure. As result there is no more bonds between the α particle on the top and the other 4 α particles (see Fig. 5). This leads to an α particle without bonding and so, to EC or to β^+ decay. The α particle splits into 3 N and 1 P (see Fig. 6).

(9) ^{55}Mn (see Table 1 and Fig. 6)

Compared to ^{55}Fe (see Fig. 5), the 3 N supplementary are bonded with 3 NPP instead of 3 × 2A bonds. The 3 N and 1 P issued from the splitting of α particle on the top of the structure are bonded with 3 NNP and 1 NP to the 4 α particles sub-structure.

(10) ^{55}Mn Fig. 2 (see Table 1 and Fig. 7)

The structure in Fig. 6 is not stable and a rearrangement is necessary. Instead of 4 NN bonds linking the 3 N and the P to the top structure in Fig. 6, there are 4 A on the Fig. 7. Moreover, the P is bonded with the 3 N and no longer with the α particles. It is now bonded alternately to 2 N (NNP/2) and to 1 N (NP/2). Therefore, the 3 NPP bonds of the 3 N supplementary on the bottom of the structure become NNP bonds.

(11) ^{55}Mn Fig. 3 (see Table 1 and Fig. 8)

There are other rearrangements within ^{55}Mn. Now, the 6 N supplementary are linked to the α particles with 6 NPP bonds. The P supplementary is also linked to α particles with 1 NPP bond and linked to two N supplementary with 2 NP/2 bonds. Nevertheless, this last link is creating one NP bond supplementary to the 14 (7 NNP) necessary.

(12) ^{55}Mn Fig. 4 (see Table and Fig. 9)

So, this NP bond supplementary, of value 4 lines is distributed between the 8A cross bonds, which become 4 NPP bonds. The number of bonds in the core is 24 and that one of the N and P supplementary is 14.

(13) ^{56}Fe (see Table 1 and Fig. 10)

The linear and cross bonds are equal to 26A. This configuration is the best possible. The 4 N are linked to the α particles with 5A, NN, NNP/2 and NPP/2 bonds.

(14) ^{56}Fe Fig. 2 (see Table 1 and Fig. 11)

To avoid the unevenness on the preceding structure, one can distribute the bonds in the way presented on Fig. 11. The interchangeability between the structures in Figs. 10 and 11 create more stability.

(15) ^{57}Fe (see Table 1 and Fig. 12)

The ^{57}Fe core structure is perfect. The 5 N supplementary are linked to the α particles with 5 NPP bonds which is an odd number. Nevertheless, the structure is stable.

(16) ^{58}Fe (see Table 1 and Fig. 13)

The core structure is perfect, the 6 N supplementary are linked to the α particles with A, NN, 2 NNP, 3 NPP bonds. In order to have 6 NPP bonds for the 6 N supplementary the number of lines in the core should be increased with 4.1875 lines. This would mean 19A, 3 NPP and NNP/2 bonds in the core instead of 26A bonds.

(17) ^{59}Fe (see Table 1 and Fig. 14)

The core structure of ^{59}Fe is perfectly shaped, as it is for ^{58}Fe. Nevertheless, there are 7 N supplementary and an excess of 9.125 lines. This configuration leads to β^- decay.

(18) ^{59}Co (see table and figures 15 and 16)

^{59}Co is the decay result of ^{59}Fe, 1 N supplementary decaying into P, the bond remaining the same (NPP). One bond of another N supplementary is consequently modified from NPP to (NN + A) bonds (see figure 15). After rearrangement, the structure looks like it does in Fig. 16.

(19) ^{60}Fe (see Table 1 and Fig. 17)

^{60}Fe structure is not stable but has a long lifetime. Nevertheless, the core structure is stable (26A) and there are 8 N supplementary linked to the α particles with 5 NPP and 3 NNP. Note the similarity with ^{57}Fe. Both ^{57}Fe and ^{60}Fe have a core structure with 26A. ^{57}Fe has 5 N supplementary linked to α particles with 5 NPP as well as ^{60}Fe, but ^{60}Fe has 3 N supplementary more linked to α particles with 3 NNP bonds.

Let us take the EB of both isotopes (AME 2016):

^{57}Fe	^{60}Fe	Difference
499.9059 MeV	525.3511 MeV	25.4452 MeV

This difference is equal to 3 × 8.4817 MeV or 3 NNP

(20) ^{61}Fe (see Table 1 and Fig. 18)

^{61}Fe is comparable to the structure ^{60}Fe. Nevertheless, the N supplementary are too numerous. So, 1 N decays rapidly into P.

4. The Author's Method of Calculation

4.1. Mind experiments

As already said (see Section 1.2.4), the author's method of calculation is not based on a theory, but rather on mind experiments. This is comparable to the work of a chemist looking for several solutions in his experiments and validating that one which fits best. Each time a new neutron or proton enters a nucleus, its binding energy value is modified by reshaping its structure among NN, NP, NNP, NPP and in case EB α values.

4.2. Examples of calculation for ^{55}Mn, ^{59}Co, and ^{63}Cu

According to the general rule (see Section 1.4) the binding energy of these three nuclei should be the following:

Experimental values of the binding energy of the three nuclei minus totals are determined in Table 2:

^{55}Mn: 482.0762 MeV (AME 2016 value) - 476.5178 MeV = 5.5584 MeV or 10 lines (for value of lines see Section 1.2.6).

^{59}Co: 517.3141 MeV (AME 2016 value) - 512.0039 MeV = 5.3102 MeV or 9.5625 lines.

^{63}Cu: 551.3847 MeV (AME 2016 value) - 547.4900 MeV = 3.8947 MeV or 7 lines.

Table 2. Theoretical binding energy values among ^{55}Mn, ^{59}Co, and ^{63}Cu.

	^{55}Mn: 12α, 6 N, 1 P		^{59}Co: 13α, 6 N, 1 P		^{63}Cu: 14α, 6 N, 1 P	
αE_{B}	EB 12α	339.9000	E_{B} 13α	368.2250	E_{B} 14α	396.5500
Core E_{B}	12 (NN+NP)	85.9332	13 (NN+NP)	93.0943	14 (NN+NP)	100.2554
N suppl. E_{B}	6 (NN+NP)	42.9666	6 (NN+NP)	42.9666	6 (NN+NP)	42.9666
P suppl. E_{B}	NPP	7.7180	NPP	7.7180	NPP	7.7180
Total (MeV)		476.5178		512.0039		547.4900

So, these lines must be added to the former calculations (see Table 2).

Results

^{55}Mn: 24A (12 NN + 12 NP) + 4 lines = 16A + 4 NPP (see Fig. 9).
12A (6 NN + 6 NP) + 6 lines = 6 NPP.
NPP = NPP.

^{59}Co: 26A (13 NN + 13 NP) = 26A (see Fig. 15).
12A (6 NN + 6 NP) + 9.5625 lines = 5A + NN + 3 NNP.
NPP = NPP.

^{59}Co: Alternative solution:
26A (13 NN + 13 NP) + 2.375 lines = 24A + NNP (see Fig. 16).
12A (6 NN + 6 NP) + 10.1250 lines = 3 NNP + 3 NPP.
NPP - 2.9375 lines = NP + NPP/2.
(2.375 + 10.125 - 2.9375 = 9.5625 lines).

^{63}Cu: 28A (14 NN + 14 NP) + 7 lines = 14A + 7 NPP (no figure).
12A (6 NN + 6 NP) = 12A.
NPP = NPP.

4.3. Decay of ^{63}Ni into ^{63}Cu

^{63}Ni decays β^- into ^{63}Cu. See Table 3, their respective binding energy structure according to the author's theory.

Comparing the two calculation results one notes the following:

(3.5 NN + 3.5 NP + 3.5 NNP) of ^{63}Ni have decayed into 7 NPP of ^{63}Cu.

In other terms (7 NN + 7 NP + 7 NNP) /2 decayed into 7 NPP.

So, the (NN + NP) bonds and NNP bonds which alternate within ^{63}Ni decay into NPP bonds within ^{63}Cu. Consequently, one N supplementary of ^{63}Ni decays into one P to create ^{63}Cu.

5. Conclusion

The distribution of binding energy in each isotope as shown above is fundamental for understanding the LENR process, and especially the transmutation process. It allows us to determine how the binding energy evolves, nucleus

Table 3. Comparison of binding energy distribution among ^{63}Ni and ^{63}Cu.

^{63}Ni	14α, 7N supplementary EB in MeV = 552.1001 Lifetime: 92 years Mode of decay: β^-				^{63}Cu	14α, 6N, 1P supplementary EB in MeV = 551.3847 Stable Nat. abundance: 69.2 %			
	14 x 28.325	396.5500	MeV			14 x 28.325	396.5500	MeV
	10.5 x 4.9365	51.8333				7 x 4.9365	34.5555	
	10.5 x 2.2246	23.3583				7 x 2.2246	15.5722	
	3.5 x 8.4818	29.6863				0 x 8.4818	0	
	0 x 7.7180	0				7 x 7.7180	54.0260	
	6 x 4.9365	29.6190				6 x 4.9365	29.6190	
	6 x 2.2246	13.3476				6 x 2.2246	13.3476	
	0 x 8.4818	0				0 x 8.4818	0	
	1 x 7.7180	7.7180				1 x 7.7180	7.7180	
		552.1125	MeV				551.3883	MeV
		+ 0.012					+ 0.004	

after nucleus, isotope after isotope. This is essential for LENR process comprehension, because the difference in the distribution of binding energy between the elements present at the beginning of the reaction and at the final stage is directly related to the energy released. Many more examples are displayed in author's book on Atomic Nuclei Binding Energy [4].

Basically, only five bond types are necessary to describe the systematics of the binding energy within the nuclei.

(1) The binding energy of the NP bond, which is the deuterium binding energy.
(2) The binding energy of the NNP bond, which the tritium binding energy.
(3) The binding energy of the NPP bond, which is the ^{3}He binding energy.
(4) The binding energy of the NN bond, which is part of the α particle binding energy.
(5) The binding energy of the α particle, which includes all the binding energies above. (Note that the PP binding energy within the α particle is equal to the binding energies of NP + NNP + NPP).

Each α particle is linked to two other α particles in a linear way and in a cross way. As well, each N supplementary is linked to two α particles by (NN + NP) bonds. These bonds could be replaced by NNP or NPP bonds. So, the stability of an element isotope is reached if the number of lines is optimal. If the number of lines is not big enough the isotope of that element decays EC or β^+ if the number of lines is too big the isotope decays β^-.

So, the examples of Fe isotopes containing 52 up to 61 nucleons presented in this paper show that:

– Fe is stable with 54, 56, 57, and 58 nucleons, i.e. 13α+ 2N, 4N, 5N, and 6N supplementary.
– Mn is stable with 55 nucleons, i.e. 12α+ 6N + 1P supplementary.
– Co is stable with 59 nucleons, i.e. 13α+ 6N +1P supplementary.

On one side the increase of the number of neutrons is linked to the strong interaction, and on the other side the transformation of one proton into one neutron (β^+ decay) or one neutron into one proton (β^- decay) is the consequence of the weak interaction.

Both decay processes are the consequence of the number of supplementary neutrons. With 2, 4, 5, and 6 neutrons Fe is stable. With fewer neutron's Fe decays β^+ into Mn. With more neutron's Fe decays β^- into Co. Nevertheless,

with three supplementary neutrons Fe decays into Mn (^{55}Mn). This is due to the equilibrium between NP and NN bonds.

^{52}Fe and ^{53}Fe have a deficit of NN bonds compared to NP bonds. ^{55}Fe suffers an excess of NN bonds (see Figs. 1, 2, 4–9). To assure the equilibrium between the bonds the number of neutrons versus protons must change, hence the β^+ decay or disintegration of an α particle into three neutrons and one proton, instead of two neutrons and two protons. Mn has 12α and Fe has 13α.

If the number of neutrons increases the number of bonds also increases. Therefore, a new equilibrium between neutrons and protons must occur. One neutron decays β^- into a proton and ^{59}Fe becomes ^{59}Co.

So, strong interaction leads to three modes of decay (β^+ and EC) if there are not enough neutrons and β^- if there are too many neutrons in order to maintain the equilibrium between NN and NP bonds, hence the importance of the A bonds as average (NN/2 + NP/2).

The valley of stability for each element is very narrow, i.e. for Fe it is limited to ^{54}Fe to ^{58}Fe and to 13α particles without any proton outside the α particles. For ^{55}Mn as well as for ^{59}Co (difference one α) there is one proton outside the α particles, and the valley of stability is limited to these two isotopes for these elements. ^{60}Co decays into ^{60}Ni, one neutron decaying β^-. This operation creates one proton more and hence one α particle more: Ni has 14α particles.

The example of Iron given in this paper is one of the numerous nuclei the author has studied. Presently, the binding energy of all isotopes of the elements up to atomic number 47 (Silver) have been determined and this work continues. Some other nuclei binding energies were also determined, especially those of atomic number from 82 to 94.

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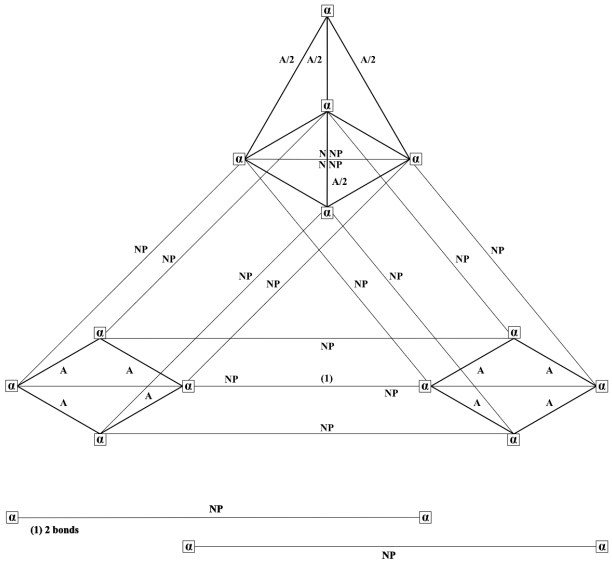
[2] Philippe Hatt, *J. Condensed Matter Nucl. Sci*. **26** (2018) 45–53, *J. Condensed Matter Nucl. Sci*. **29** (2019) 260–274.
[3] The AME 2016 atomic mass evaluation.
[4] www.philippehatt.com.

Appendix A. $^{52}_{26}$Fe

Structure: 13 α, 0 N, 0 P
Linear and cross bonds: 10A, 12 NP, 2 NNP
N supplementary bonds: 0



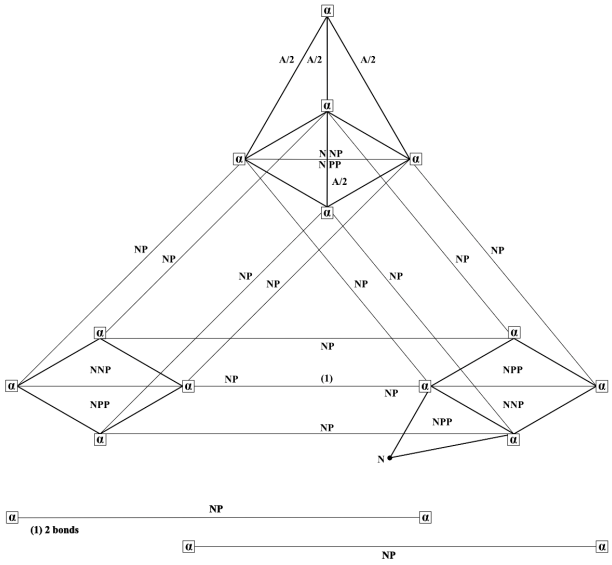
h

$^{52}_{26}$Fe	13α, 0N, 0P supplementary	E_B (MeV)=447.6978
Lifetime=8.2 h Mode of decay: β^-	$\left\{\begin{array}{lll} 13 & \times & 28.3250 \\ 6.5-1.5 & \times & 4.9365 \\ 6.5+10.5 & \times & 2.2246 \\ 2 & \times & 8.4818 \\ 0 & \times & 7.7180 \end{array}\right\}$	$368.2250\ MeV$ 24.685 37.8182 16.9636 0
	$\left\{\begin{array}{lll} 0 & \times & 4.9365 \\ 0 & \times & 2.2246 \\ 0 & \times & 8.4818 \\ 0 & \times & 7.7180 \end{array}\right\}$	0 0 0 0
		447.6893 MeV -0.009

Figure 1. Binding energy distribution among ^{52}Fe .

Appendix B. $^{53}_{26}$Fe

Structure: 13 α, 1 N, 0 P
Linear and cross bonds: 2 A, 12 NP, 3 NNP, 3 NPP
N supplementary bonds: NPP

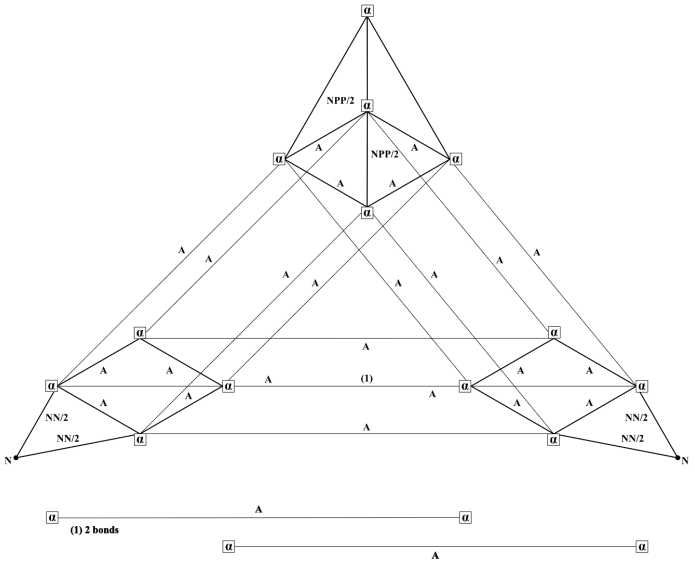


$^{53}_{26}$Fe	13α, 1N, 0P supplementary	E_{B} (MeV)=458.3863
Lifetime=8.5 min Mode of decay: β^+, EC	$\left\{\begin{array}{lll} 13 & \times & 28.3250 \\ 6.5-5.5 & \times & 4.9365 \\ 6.5+10.5 & \times & 2.2246 \\ 3 & \times & 8.4818 \\ 3 & \times & 7.7180 \end{array}\right\}$	368.2250 MeV 4.9365 28.9198 25, 4454 23.1540
	$\left\{\begin{array}{lll} 0 & \times & 4.9365 \\ 0 & \times & 2.2246 \\ 0 & \times & 8.4818 \\ 0 & \times & 7.7180 \end{array}\right\}$	0 0 0 7.7180
		458.3987 MeV +0.012

Figure 2. Binding energy distribution among ^{53}Fe.

Appendix C. $^{54}_{26}$Fe

Structure: 13 α, 2 N, 0 P
Linear and cross bonds: 24 A, NPP
N supplementary bonds: 2 NN

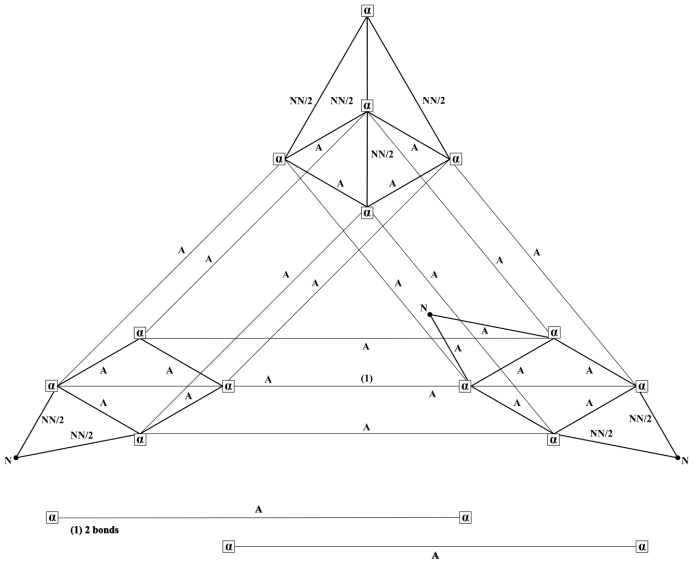


$^{54}_{26}$Fe	13α, 2 N, 0P supplementary	E_B (MeV)=471.7646
Stable Nat, abundance: 5.8 %	$\left\{\begin{array}{lll} 13 & \times & 28.3250 \\ 6.5-5.5 & \times & 4.9365 \\ 6.5+5.5 & \times & 2.2246 \\ 0 & \times & 8.4818 \\ 1 & \times & 7.7180 \end{array}\right\}$	368.2250 MeV 4.9365 59.2380 26.69524454 0
	$\left\{\begin{array}{lll} 2 & \times & 4.9365 \\ 0 & \times & 2.2246 \\ 0 & \times & 8.4818 \\ 0 & \times & 7.7180 \end{array}\right\}$	9.8730 0 0 0
		471.7492 MeV -0.015

Figure 3. Binding energy distribution among ^{54}Fe.

Appendix D. $^{55}_{26}$Fe

Structure: 13 α, 3 N, 0 P
Linear and cross bonds: 24 A, 2 NN
N supplementary bonds: 2 A, 2 NN

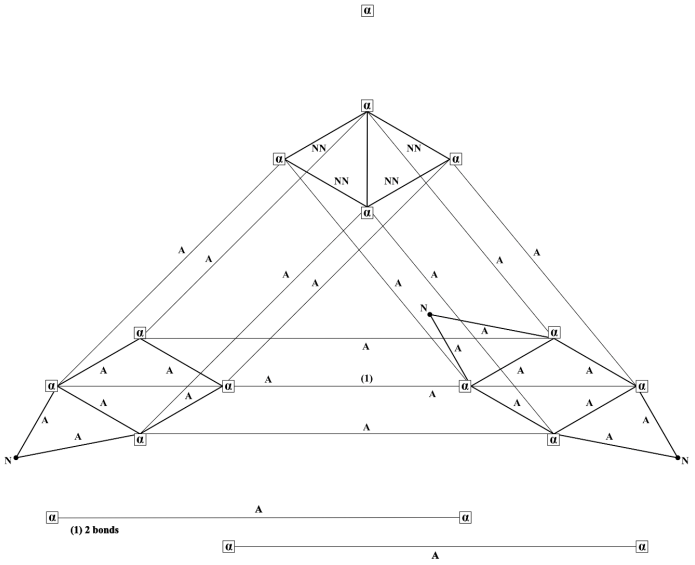


$^{55}_{26}$Fe	13α, 3 N, 0P supplementary	E_{B} (MeV)=481.0627
Lifetime = 2.6 years Mode of decay: EC	$\left\{\begin{array}{lll} 13 & \times & 28.3250 \\ 6.5-7.5 & \times & 4.9365 \\ 6.5+5.5 & \times & 2.2246 \\ 0 & \times & 8.4818 \\ 0 & \times & 7.7180 \end{array}\right\}$	368.2250 MeV 69.1110 26.6952 0 0
	$\left\{\begin{array}{lll} 3 & \times & 4.9365 \\ 1 & \times & 2.2246 \\ 0 & \times & 8.4818 \\ 0 & \times & 7.7180 \end{array}\right\}$	14.8095 2.2246 0 0
		$\overline{481.0653\ \text{MeV}}$ +0.003

Figure 4. Binding energy distribution among ^{55}Fe.

Appendix E. $^{55}_{26}$Fe Fig. 2

Structure: 13 α, 3 N, 0 P
Linear and cross bonds: 20 A, 4 NN
N supplementary bonds: 6 A

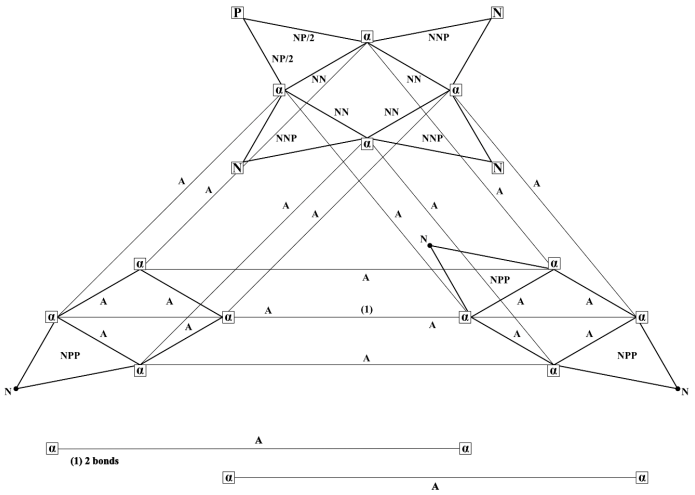


$^{55}_{26}$Fe Fig.2	13α, 3 N, 0P supplementary	E_{B} (MeV)=481.0627
Lifetime = 2.6 years Mode of decay: EC	$\left\{\begin{array}{lll} 13 & \times & 28.3250 \\ 6.5-7.5 & \times & 4.9365 \\ 6.5+3.5 & \times & 2.2246 \\ 0 & \times & 8.4818 \\ 0 & \times & 7.7180 \end{array}\right\}$	368.2250 MeV 69.1110 22.2460 0 0
	$\left\{\begin{array}{lll} 3 & \times & 4.9365 \\ 3 & \times & 2.2246 \\ 0 & \times & 8.4818 \\ 0 & \times & 7.7180 \end{array}\right\}$	14.8095 6.6738 0 0
		$\overline{481.0653\ \text{MeV}}$ +0.003

Figure 5. Binding energy distribution among ^{55}Fe.

Appendix F. $^{55}_{25}$Mn

Structure: 12 α, 6 N, 1 P
Linear and cross bonds: NP, 3 NNP, 3 NPP

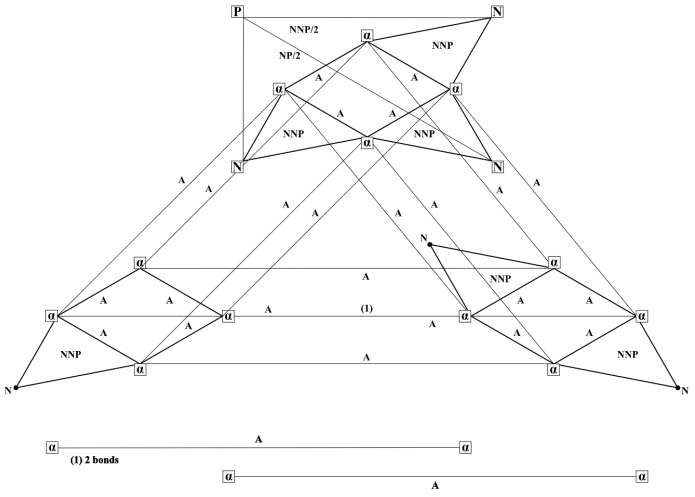


$^{55}_{25}$Mn	12α, 6 N, 1P supplementary	E_B (MeV)=482.0762
	$\left\{\begin{array}{lll} 12 & \times & 28.3250 \\ 6.5-7.5 & \times & 4.9365 \\ 6.5+3.5 & \times & 2.2246 \\ 0 & \times & 8.4818 \\ 0 & \times & 7.7180 \end{array}\right\}$	339.9000 MeV 69.1110 22.2460 0 0
Stable Nat. abundance 100%	$\left\{\begin{array}{lll} 0 & \times & 4.9365 \\ 1 & \times & 2.2246 \\ 3 & \times & 8.4818 \\ 3 & \times & 7.7180 \end{array}\right\}$	0 2.2246 25.4454 23.1540
		482.0810 MeV +0.005

Figure 6. Binding energy distribution among ^{55}Mn.

Appendix G. $^{55}_{25}$Mn Fig. 2

Structure: 12 α, 6 N, 1 P
Linear and cross bonds: 24 A
N, P supplementary bonds: NP/2, 6.5 NNP

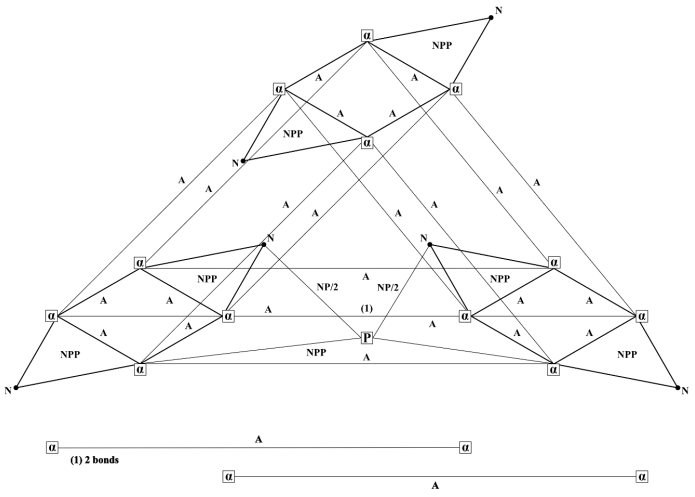


$^{55}_{25}$Mn Fig. 2	12α, 6 N, 1P supplementary	E_B (MeV)=482.0762
Stable Nat. abundance 100%	$\left\{\begin{array}{lll} 12 & \times & 28.3250 \\ 6+6 & \times & 4.9365 \\ 6+6 & \times & 2.2246 \\ 0 & \times & 8.4818 \\ 0 & \times & 7.7180 \end{array}\right\}$	339.9000 MeV 59.2380 26.6952 0 0
	$\left\{\begin{array}{lll} 0 & \times & 4.9365 \\ 1 & \times & 2.2246 \\ 3 & \times & 8.4818 \\ 3 & \times & 7.7180 \end{array}\right\}$	0 1.1123 55.1317 0
		482.0772 MeV +0.001

Figure 7. Binding energy distribution among ^{55}Mn.

Appendix H. $^{55}_{25}$Mn Fig. 3

Structure: 12 α, 6 N, 1 P
Linear and cross bonds: 24 A
N, P supplementary bonds: NP, 7 NNP

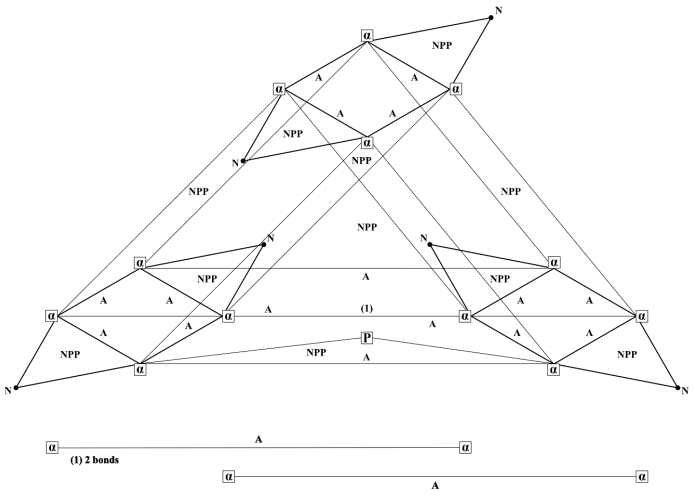


$^{55}_{25}$Mn Fig. 3	12α, 6 N, 1P supplementary	E_B (MeV)=482.0762
Stable Nat. abundance 100%	$\left\{\begin{array}{lll} 12 & \times & 28.3250 \\ 6+6 & \times & 4.9365 \\ 6+6 & \times & 2.2246 \\ 0 & \times & 8.4818 \\ 0 & \times & 7.7180 \end{array}\right\}$	339.9000 MeV 59.2380 26.6952 0 0
	$\left\{\begin{array}{lll} 0 & \times & 4.9365 \\ 1 & \times & 2.2246 \\ 0 & \times & 8.4818 \\ 7 & \times & 7.7180 \end{array}\right\}$	0 2.2246 0 54.0260
		482.0260 MeV +0.008

Figure 8. Binding energy distribution among ^{55}Mn.

Appendix I. $^{55}_{25}$Mn Fig. 4

Structure: 12 α, 6 N, 1 P
Linear and cross bonds: 16 A, 4 NPP
N, P supplementary bonds: 7 NPP

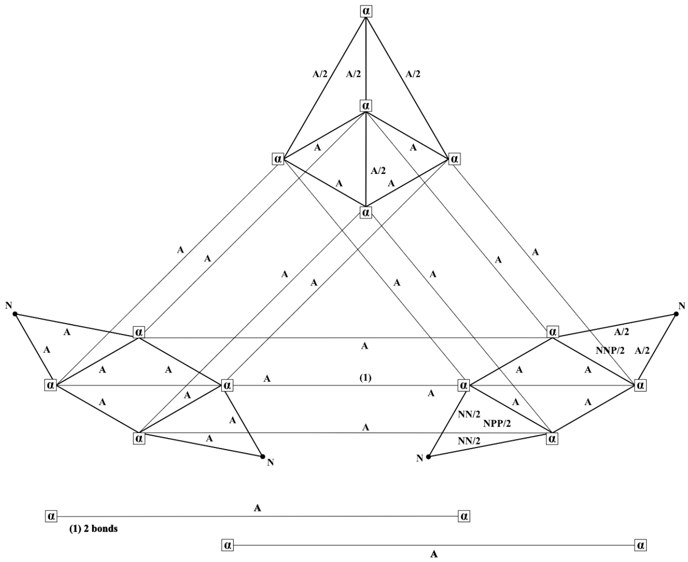


$^{55}_{25}$Mn Fig. 4	12α, 6 N, 1P supplementary	E_B (MeV)=482.0762
Stable Nat. abundance 100%	$\left\{\begin{array}{lll} 12 & \times & 28.3250 \\ 6+2 & \times & 4.9365 \\ 6+2 & \times & 2.2246 \\ 0 & \times & 8.4818 \\ 4 & \times & 7.7180 \end{array}\right\}$	339.9000 MeV 39.4920 17.7968 0 30.8720
	$\left\{\begin{array}{lll} 0 & \times & 4.9365 \\ 0 & \times & 0 \\ 0 & \times & 8.4818 \\ 7 & \times & 7.7180 \end{array}\right\}$	0 2.2246 0 54.0260
		482.0868 MeV +0.011

Figure 9. Binding energy distribution among ^{55}Mn.

Appendix J. $^{56}_{26}$Fe

Structure: 13 α, 4 N, 0 P
Linear and cross bonds: 26 A
N, P supplementary bonds: 5 A, NN, NNP/2, NPP/2

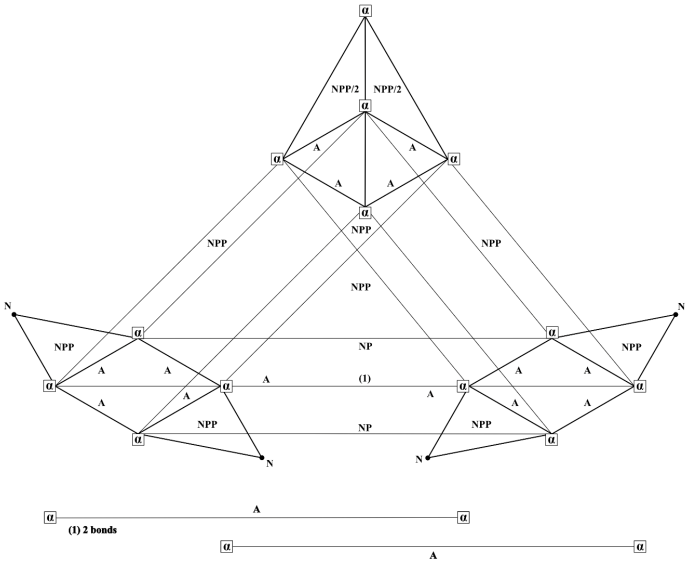


$^{56}_{26}$Fe	13α, 4 N, 0 P supplementary	E_B (MeV)=492.2598
	13 × 28.3250	368.2250 MeV
	6.5 + 6.5 × 4.9365	64.1745
Stable	6.5 + 6.5 × 2.2246	28.9198
Nat. abundance	0 × 8.4818	0
91.8%	0 × 7.7180	0
	3.5 × 4.9365	17.2778
	2.5 × 0	5.5615
	0.5 × 8.4818	4.2409
	0.5 × 7.7180	3.8590
		492.2585 MeV
		-0.001

Figure 10. Binding energy distribution among ^{56}Fe.

Appendix K. $^{56}_{26}$Fe Fig.2

Structure: 13 α, 4 N, 0 P
Linear and cross bonds: 14 A, 2 NP, 5 NPP
N, P supplementary bonds: 4 NPP

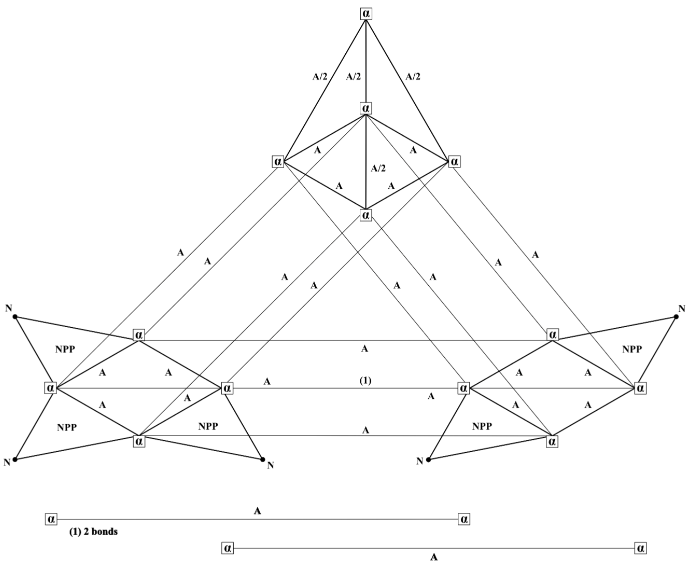


$^{56}_{26}$Fe Fig.2	13α, 4 N, 0 P supplementary	E_{B} (MeV)=492.2598
Stable Nat. abundance 91.8%	$\left\{\begin{array}{lll} 13 & \times & 28.3250 \\ 6.5+0.5 & \times & 4.9365 \\ 6.5+2.5 & \times & 2.2246 \\ 0 & \times & 8.4818 \\ 5 & \times & 7.7180 \end{array}\right\}$	368.2250 MeV 34.5555 20.0214 0 38.5900
	$\left\{\begin{array}{lll} 0 & \times & 4.9365 \\ 0 & \times & 2.2246 \\ 0 & \times & 8.4818 \\ 5 & \times & 7.7180 \end{array}\right\}$	0 0 0 30.8720
		$\overline{492.2639}$ MeV +0.004

Figure 11. Binding energy distribution among ^{56}Fe.

Appendix L. $^{57}_{26}$Fe

Structure: 13 α, 5 N, 0 P
Linear and cross bonds: 26 A
N, P supplementary bonds: 5 NPP

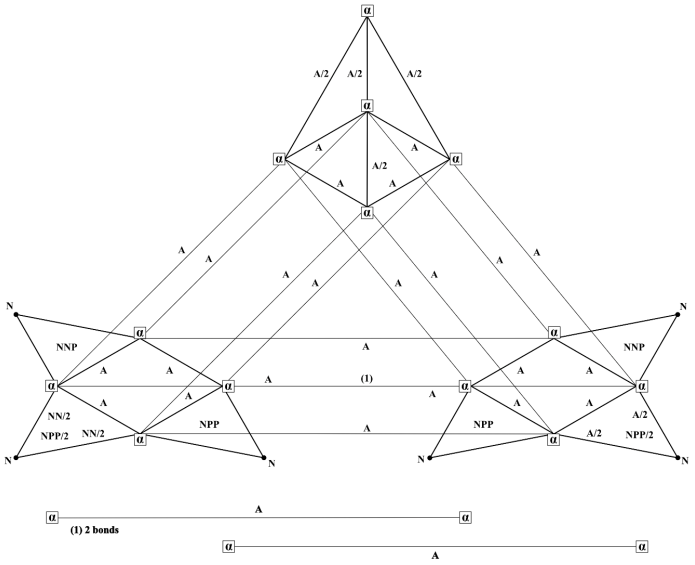


$^{57}_{26}$Fe	13α, 5 N, 0 P supplementary	E_{B} (MeV)=499.9059
	13×28.3250	368.2250 MeV
	$6.5+6.5 \times 4.9365$	64.1745
Stable	$6.5+6.5 \times 2.2246$	28.9198
Nat. abundance	0×8.4818	0
2.1%	0×7.7180	0
	0×4.9365	0
	0×2.2246	0
	0×8.4818	0
	5×7.7180	38.5900
		$\overline{499.9093}$ MeV
		+0.004

Figure 12. Binding energy distribution among ^{57}Fe.

Appendix M. $^{58}_{26}$Fe

Structure: 13 α, 6 N, 0 P
Linear and cross bonds: 26 A
N, P supplementary bonds: A, NN, 2 NP, 3 NPP

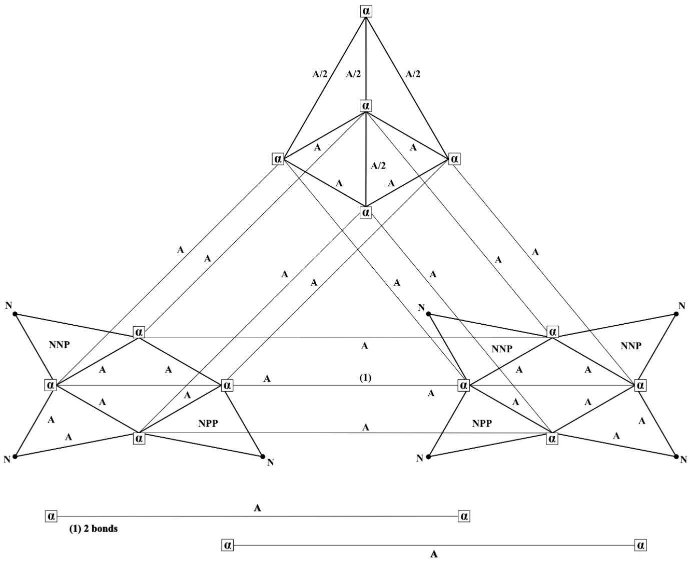


$^{58}_{26}$Fe	13α, 6 N, 0 P supplementary	E_B (MeV)=509.9505
Stable Nat. abundance 0.3%	$\left\{\begin{array}{lll} 13 & \times & 28.3250 \\ 6.5+6.5 & \times & 4.9365 \\ 6.5+6.5 & \times & 2.2246 \\ 0 & \times & 8.4818 \\ 0 & \times & 7.7180 \end{array}\right\}$	368.2250 MeV 64.1745 28.9198 0 0
	$\left\{\begin{array}{lll} 1.5 & \times & 4.9365 \\ 0.5 & \times & 2.2246 \\ 2 & \times & 8.4818 \\ 3 & \times & 7.7180 \end{array}\right\}$	7.4048 1.1123 16.9636 23.1540
		$\overline{509.9540\ \text{MeV}}$ +0.003

Figure 13. Binding energy distribution among ^{58}Fe.

Appendix N. $^{59}_{26}$Fe

Structure: 13 α, 7 N, 0 P
Linear and cross bonds: 26 A
N, P supplementary bonds: 4 A, 3 NNP, 2 NPP

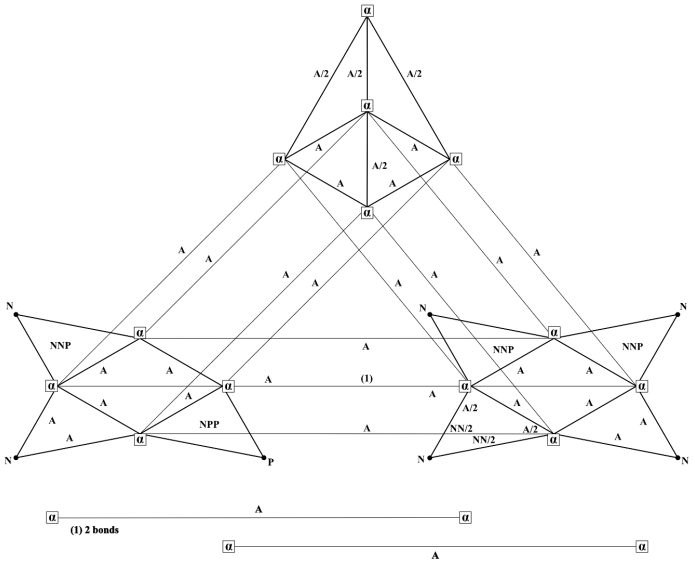


$^{59}_{26}$Fe	13α, 7 N, 0 P supplementary	E_{B} (MeV)=516.5315
	13×28.3250	368.2250 MeV
	$6.5 + 6.5 \times 4.9365$	64.1745
Lifetime:	$6.5 + 6.5 \times 2.2246$	28.9198
45.1 days	0×8.4818	0
Mode of decay: β^-	0×7.7180	0
	2×4.9365	9.8730
	2×2.2246	4.4492
	3×8.4818	25.4454
	2×7.7180	15.4360
		516.5229 MeV
		-0.008

Figure 14. Binding energy distribution among ^{59}Fe.

Appendix O. $^{59}_{27}$Co

Structure: 13 α, 6 N, 1 P
Linear and cross bonds: 26 A
N, P supplementary bonds: 5 A, NN, 3 NNP, 1 NPP

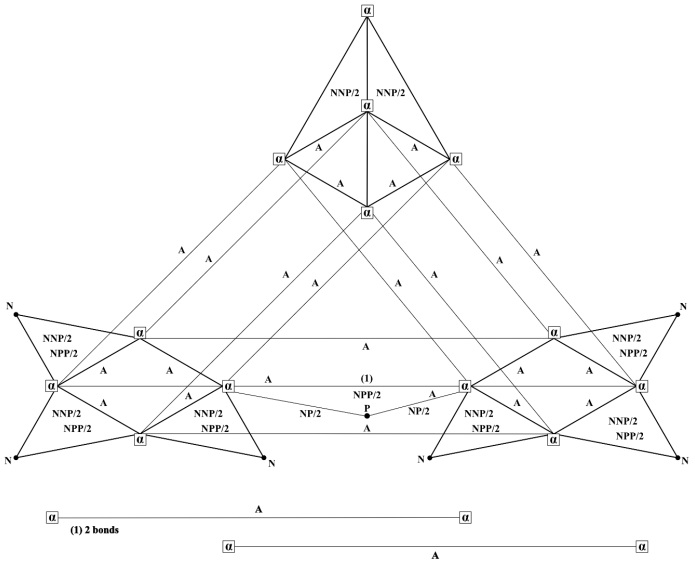


$^{59}_{27}$Co	13α, 6 N, 1 P supplementary	E_B (MeV)=517.3141
	$\left\{\begin{array}{lll} 13 & \times & 28.3250 \\ 6.5+6.5 & \times & 4.9365 \\ 6.5+6.5 & \times & 2.2246 \\ 0 & \times & 8.4818 \\ 0 & \times & 7.7180 \end{array}\right\}$	368.2250 MeV 64.1745 28.9198 0 0
Stable Nat. abundance 100%	$\left\{\begin{array}{lll} 3.5 & \times & 4.9365 \\ 2.5 & \times & 2.2246 \\ 3 & \times & 8.4818 \\ 1 & \times & 7.7180 \end{array}\right\}$	17.2778 5.5615 25.4454 7.7180
		517.3220 MeV +0.008

Figure 15. Binding energy distribution among ^{59}Co.

Appendix P. $^{60}_{26}$Co Fig. 2

Structure: 13 α, 6 N, 1 P
Linear and cross bonds: 26 A, NNP
N, P supplementary bonds: NP, 3 NNP, 3 NNP, 3.5 NPP

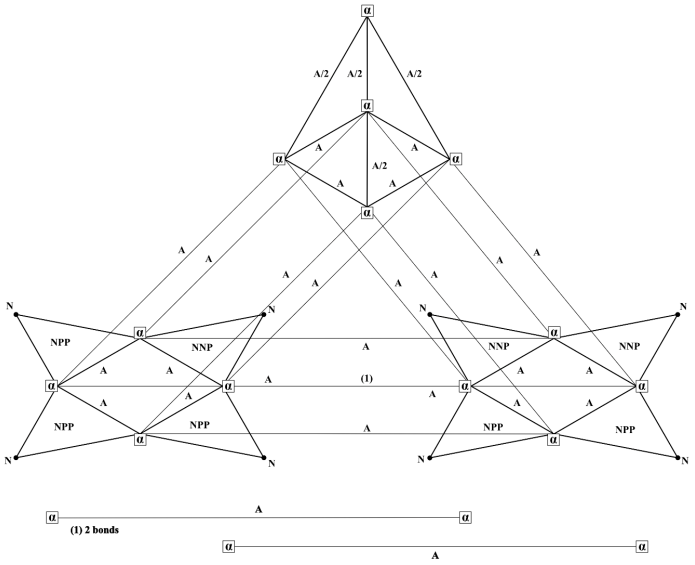


$^{59}_{27}$Co Fig. 2	13α, 6 N, 1 P supplementary	E_{B} (MeV)=517.3141
	$\left\{\begin{array}{lll} 13 & \times & 28.3250 \\ 6.5+5.5 & \times & 4.9365 \\ 6.5+5.5 & \times & 2.2246 \\ 1 & \times & 8.4818 \\ 0 & \times & 7.7180 \end{array}\right\}$	368.2250 MeV 59.2380 26.6952 8.4818 0
Stable Nat. abundance 100%	$\left\{\begin{array}{lll} 0 & \times & 4.9365 \\ 1 & \times & 2.2246 \\ 3 & \times & 8.4818 \\ 3.5 & \times & 7.7180 \end{array}\right\}$	0 2.2846 25.4454 27.0130
		517.3230 MeV -0.009

Figure 16. Binding energy distribution among ^{59}Co.

Appendix Q. $^{60}_{26}$Fe

Structure: 13 α, 8 N, 0 P
Linear and cross bonds: 26 A
N, P supplementary bonds: NP, 3 NNP, 5 NPP

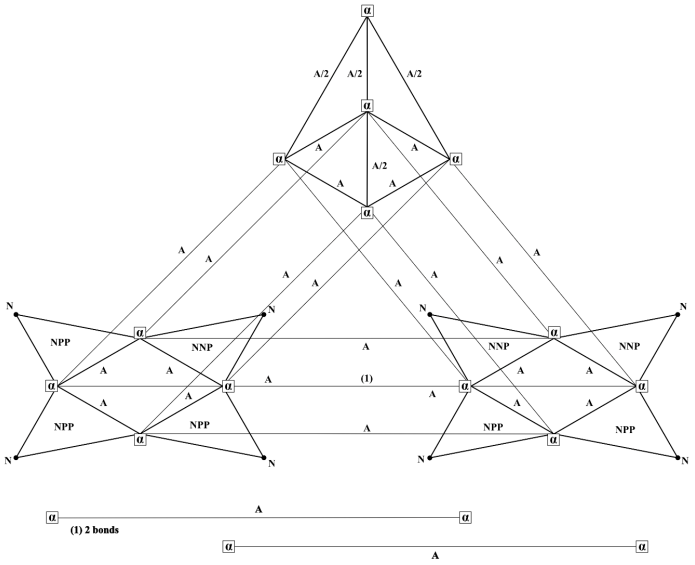


$^{60}_{26}$Fe	13α, 8 N, 0 P supplementary	E_B (MeV)=525.3511
Lifetime: 3×10^5 years Mode of decay: β^-	13×28.3250	368.2250 MeV
	$6.5 + 6.5 \times 4.9365$	64.1745
	$6.5 + 6.5 \times 2.2246$	28.9198
	0×8.4818	0
	0×7.7180	0
	0×4.9365	0
	0×2.2246	0
	3×8.4818	25.4454
	5×7.7180	38.5900
		525.3547 MeV
		+0.004

Figure 17. Binding energy distribution among ^{60}Fe.

Appendix R. $^{61}_{26}$Fe

Structure: 13 α, 9 N, 0 P
Linear and cross bonds: 26 A
N, P supplementary bonds: 5 A, 2 NNP, 4.5 NPP



$^{61}_{26}$Fe	13α, 9 N, 0 P supplementary	E_B (MeV)=530.9298
Lifetime: 6 min Mode of decay: β^-	$\left\{\begin{array}{lll} 13 & \times & 28.3250 \\ 6.5+6.5 & \times & 4.9365 \\ 6.5+6.5 & \times & 2.2246 \\ 0 & \times & 8.4818 \\ 0 & \times & 7.7180 \end{array}\right\}$	368.2250 MeV 64.1745 28.9198 0 0
	$\left\{\begin{array}{lll} 2.5 & \times & 4.9365 \\ 2.5 & \times & 2.2246 \\ 2 & \times & 8.4818 \\ 4.5 & \times & 7.7180 \end{array}\right\}$	12.3413 5.5615 16.9636 34.7310
		530.9167 MeV -0.013

Figure 18. Binding energy distribution among ^{61}Fe.