J. Condensed Matter Nucl. Sci. 13 (2014) 234–241

Research Article

# Bose–Einstein Condensation and Inverted Rydberg States in Ultra-high Density Deuterium Clusters Related to Low Energy Nuclear Reactions

Heinrich Hora*

*Department of Theoretical Physics, University of New South Wales, Sydney 2052, Australia*

George H. Miley and Xiaoling Yang

*Department of Nuclear, Plasma and Radiological Engineering, University of Illinois, Urbana, IL 61801, USA*

**Abstract**

Results about low energy nuclear reactions (LENR) are related to very high density clusters of deuterons where properties of Bose–Einstein condensation and/or inverted Rydberg states are compared. A modification of Bohr's atom model is used to overcome the problem that a quantum state with $n = 1$ does not emit radiation from an "orbiting" electron. This permits then the description of the inverted state of Rydberg matter in agreement with the recent measurements of Holmlid et al. for deuterium clusters with ultra-high deuteron densities in the range of $10^{29}\text{cm}^{-3}$. A virtual oscillation model for laser excitation clusters explains the low intensity ionization threshold in clusters. MeV particle emission from LENR can then be compared with measurements from inverted Rydberg states.


*Keywords:* Bose–Einstein condensation, Generalized Bohr model, Low energy nuclear reactions, Rydberg matter, Ultra-high density deuterium clusters

## 1. Introduction

Ultra-high density states of deuteron clusters are considered which generation in empty crystal positions (Schottky defects) were measured [1]. Similar ultra-high density states of deuteron clusters in surface defects were measured [2–4] where the distance between the deuterons in these states was measured from deuteron emission using mass spectrometry. The distances between the deuterons within the clusters in the range of 2 pm were explained as the result of inverted Rydberg states. These states may be compared with Bose–Einstein-Condensations (BEC) [1,2,5,6]. This was studied in connection with low energy nuclear reactions (LENR) beginning with a two body Bose–Einstein

*E-mail: h.hora@unsw.edu.au

© 2014 ISCMNS. All rights reserved. ISSN 2227-3123

mechanism [7] with general consequences [8]. The essential property for BEC was opened by the very small deuteron distance of 2 pm for nuclear reactions with probability times in the kilosecond range [9] which could be concluded from experiments with plasma loading processes in palladium grains [10] even before the convincing LENR processes were discovered [11,12]. This all was leading to the recent significant progress about LENR [13] using gas loading of deuterium in palladium nano-grains.

The interest in the deuteron interaction processes in the 2 pm distance range needs a comparison with the different properties of the involved models for explaining the phenomenon and may finally lead to an interaction process where a virtual oscillation model of quantum states is applied.

## 2. Basic Experimental Proof of LENR

In view of the complex developments of low temperature nuclear reactions of deuterons in gaseous-plasma loaded palladium crystals [10,13], it should first be underlined on what crucial experimental result LENR [11,12] is based. It was the discovery that the deuterium loaded palladium is generating nuclei up to gold and higher proton numbers $Z$ with nucleon numbers $A$. Thanks to the very sophisticated diagnostics facilities of the Frederick Seitz center at the University of Illinois at Urbana, IL, the generation probability $P(Z)$ could be measured where the maxima and minima were similar to the element distribution in the Universe [14]. The large scale minimum minimum at $A$ around 155 could be compared with the minimum of the uranium fission products at $A$ of 119 [6]. While this large scale minimum is standard knowledge for fission of unexcited uranium nuclei, it was discovered [15,16], that a difference occurs if the fissioning uranium is in an excited state of up to MeV energy. The $P(Z)$ curve has then a little local maximum peak at the large scale minimum. It is very surprising that this Maruhn–Greiner-maximum at uranium fission [15] was also measured for LENR at $A$ of 155. This fact is a very convincing experimental proof of LENR.

After this solid experimental fact, it could be speculated how this result may be understood. It could be suggested, that LENR may be via a compound nuclear reaction [17] where indeed a very hypothetical nucleus with $A = 310$ may be involved. What was interesting, is that the new model for derivation of the nuclear magic numbers [14] was possible from the few maximum curves of $P(Z)$ similar to to the astrophysics case, where higher magic numbers above 126 were 184 and further. Both were fitting a double magic number nucleus with $A = 310$ [5,6] as a initial hypothetical result.

## 3. Two Picometer Deuteron Distance

The significance of a 2 ps deuteron distance was initially based on an evaluation [9] of gas phase loading of deuterium in palladium crystals [10]. The reduction of Coulomb screening by a factor 5 is well known in high temperature plasmas [18]. Evaluating the semi-empirical DD reaction probability for the usual hot fusion, for muon-catalized fusion and what could be concluded from deuterium loaded palladium [9] was that the Coulomb screening is resulting in a factor 13 for the deuterons within the crystal, in which case the DD reaction probability is given by a time $t*$ of kiloseconds (ks) to nearly Ms and the distance of the reacting deuterons are in the range $r*$ of about 2 pm. It is remarkable that the K-shell capture of electrons at the inverse radioactive beta-decay have similar ranges $t*$ and $r*$.

After this semi-empirical derivation of the screening factor 13, this was derived in a detailed quantum mechanical study by Czerski, Huke et al. [19,20]. This means that the deuterons are screened like neutral particles, e.g. like neutrons, moving within the palladium lattice like spheres of 2 pm diameter following Maxwell statistics. For smaller diameters the screening of the deuterons is lost and then electrical repulsion will occur. These screened deuterons may then cling together by attractive forces where the Casimir effect is a possibility. But it well can be that these clusters have the properties of inverted Rydberg states as concluded from the measured 630 eV emission of deuterons as a kind of Coulomb explosion [2]. In contrast to larger distances [7,8], the 2 pm distances of deuterons in clusters may also lead to the description as Bose–Einstein condensation. This state is ideal to understand the nonlocality of the deuterons within the clusters. More than 100 deuterons can then be in a cluster of about 10 pm diameter moving around within the palladium lattice like cold neutrons. If one cluster moves to a distance of about 2 pm to a palladium nucleus, all the deuterons in the cluster can react as if in 2 pm distance due to the quantum property of BEC within the cluster. A compound nuclear reaction could then be [5,6]

$$^{108}\mathrm{Pd}_{46} + 164\mathrm{D} = {}^{310}\mathrm{X}_{126} + 42\ {}^{3}\mathrm{He}_2 + E, \tag{1}$$

where $E$ is the energy release. The mass per nucleon expressed in proton masses in X is derived to be 1.004946 (ignoring the minor contribution associated with $E$). This compares favourably with the value of uranium 1.0001868 or with the very low value comparing favourably with iron of 0.9988376.

## 4. Discussion about the BEC Model in View of Inverted Rydberg States

The here presented BEC model is one possible model with several open questions in order to explain the clear fact of the Maruhn–Greiner result in Miley's LENR measurement. What has not yet been solved is, e.g., what happens with the screening electrons in their Fermi–Dirac background within the ultra-high density clusters. The screened deuterons are not really neutral atoms or molecules. A contrast may be the inverted Ryberg state for the clearly measured ultra-high density deuterium in the clusters for which case the following modification of Bohr's atom model may be given.

The alternative approach to clusters in void-defects of solids was the experimental discovery of cluster states with ultra-high deuterium densities of up to $10^{29}$ per $\mathrm{cm}^3$ in near-surface crystal defects of iron oxide. This was found [2] from measurements of the emission of 630 eV deuterium ions during laser irradiation with the conclusion that these clusters are in an inverted Rydberg state. It may be assumed that the clusters in these surface defects are related to the ones we have described relative to the defects formed by loading-unloading methods [1].

The generation of Rydberg matter in the universe as interstellar clusters has been discussed over recent years. These are molecular structures where the electrons are in orbital states with a orbital quantum number $m = 1$ or higher. It is difficult to produce these states in the laboratory because the atoms needed an energetic excitation much higher than the molecular binding energy for joining to a molecule. However, the statistics in interstellar space does permit this. Such RM clusters in space cannot be detected by spectroscopic methods but may be dark matter which only are measured by gravitation. Indeed, this may be the dark matter predicted to have some concentration inside and near the discs of galaxies.

A special method was developed by Holmlid et al. [2–4] to use the catalytic property at the surface of ion oxide for producing Rydberg states. Then within voids of the crystal defects it is possible, that hydrogen or deuterium atoms obtain an $m = 1$ excitation, leading to a RM molecular compound catalytically without counterproductive higher energies as in vacuum. It is calculated [2] that the proton distance of 74 pm in a H–H molecule with valent binding is changed into a distance

$$d = 150 \text{ pm} \tag{2}$$

in the RM state. This state is a metallic modification of hydrogen H(1) where the number one expresses the $m = 1$ state.

The inverted RM is using an inversion of the role of the electron and proton to form a hydrogen atom. The normal atom is based on the electric field of a central proton and the electron is attracted by this field. In Bohr's model, the electron is considered in a point-mechanical treatment as rotating with an anglar momentum $p$ at a radius $r$ such that the quantum relation

$$rp = nh/2\pi = \hbar \quad (n = 1, 2, 3 \ldots) \tag{3}$$

is fulfilled. This rotation is not true in the ground state for $n = 1$ because the electron would then emit energy by radiation. This problem was solved by Schrödinger's quantum mechanics where the electron orbiting is possible only for $n = 2$ and higher. The merit of both models should not be ignored. The transition of a rotating electron from a higher to lower orbit described the radiation emission to arrive at the measured times of about $10^{-8}$ s for spontaneous emission. This was difficult yo be derived in quantum mechanics and was achieved only by Dirac's introduction of the quantization of the electromagnetic field energy density (second quantization).

The following quantum mechanical modification of Bohr's semiclassical atom model is possible in the following way in order to overcome the orbiting problem for $n = 1$. The electric field energy gained by an electron when falling into the proton depends on the distance $r$ and can be compared with the Fermi–Dirac energy for squeezing the electron into a sphere of radius $r$. The difference of the exponents of the energies arrives at a radius where both energies are the same. This is just the value [21]

$$r_{\mathrm{B}} = \hbar^2/me^2 \tag{4}$$

of the Bohr radius where $m$ is the rest mass and $e$ the electric charge of an electron (see Section 2.3 of [21]). Using this model, the measured polarization shift of spectral lines in plasma (Inglis–Teller effect) for hydrogen can be theoretically explained [22] with higher accuracy than by the earlier derived model by Griem.

The inverted hydrogen atom occurs when an electron produced the central electric field and the proton (or deuteron) falls into the electron until a radius is reached where the electric field energy gained is equal to the increase of the Fermi–Dirac quantum energy. The radius is then different from the normal Rydeberg case by the square root of the ratio of the mass of the proton p or deuteron D, see the equations between 2.16 and 2.17 of Ref. [22]. Instead of the distance $d$, Eq. (2), in a Rydberg cluster, the distance $d*$ in the inverted deuterium RM cluster D(-1) is then expressed with the deuteron mass $m_D$

$$d^* = (m/m_{\mathrm{D}})^{1/2} d. \tag{5}$$

Remarkably, this value is 2.5 pm as was initially calculated [2] from the orbital motion within the clusters.

The direct experimental proof of this distance was obtained from measurements using laser irradiation of the catalytic produced D(-1) clusters [2]. The mechanism can be seen when photons from a laser beam irradiate the cluster and photo-electrically removing electrons at binding centers for the deuterons in the inverted RM. The remaining deuterons are then to repel each other by Coulomb repulsion causing a Columbic Explosion (CE) and are subsequently emitted into the vacuum above the iron oxide. Time of Flight (TOF) measurements show energies of 630 eV. This exactly corresponds to an initial distance of the deuterons in the cluster of

$$d^*_{\mathrm{exo}} = 2.3 \text{ pm}. \tag{6}$$

The quantum mechanical explanation of the laser produced electron emission process is given with fitting the parameters obtained in this experiment. The threshold of the laser intensity is found to be close to or a little higher than [23]

$$I_{\mathrm{three}} = 10^{10}\,\mathrm{W/cm^2} \tag{7}$$

for a wave length of 565 pm. It should be noted that this experiment did not work with H(1) and H(-1) RM at the low intensities (7). This indicates that the D(-1) clusters have properties of BEC which is not normally possible with protons as fermions while deuteron are bosons. As suggested by Kim [8], at the densities involving pseudo bosons, this may result form allowing condensation.

## 5. Oscillation Model for Quantum States Analoque to Free Electrons

Low energy laser excitation may be another way of exciting LENR reactions in RM states. This requires development of efficient methods for coupling the laser photon energy with the inverted RM state of D(-1) clusters in order to remove electrons, and cause the D-atom in the cluster to fuse. The mechanisms to explain removal of the electrons by the laser field from the inverted surface RM state of D(-1) clusters is explained from the oscillation of the electron in an electromagnetic field (quiver motion at laser irradiation) in combination with a quantum relation to obtain a correspondence principle for the electromagnetic interaction [24].

A free electron in space, e.g. in a plasma, quivers in a laser field with the amplitude $E$ of the laser field having a maximum elongation

$$r = eE/m\omega^2 \tag{8}$$

with a maximum momentum $p = mv$ from the quiver velocity $v$

$$p = mv = eE/\omega, \tag{9}$$

where $\omega$ is the radian frequency of the laser. Indeed, free electrons in vacuum perform the quiver motion as detected also from Thomsen scattering where the oscillation energy of the electrons

$$\varepsilon_{\text{osc}} = (eE/\omega)^2/2m \tag{10}$$

is the quiver energy.

A basic difference between this analysis as classical point mechanical motion in vacuum occurs when the electron is bound in an unexcited hydrogen atom or any bound state including the Rydberg state. If the laser field has a sufficiently high intensity $I > I_\text{i}$ above the ionization threshold $I_\text{i}$, in vacuum the electron will be ionized and the motion of the electron will then follow the quiver motion where $I_\text{i}$ is defined by the ionization energy $\varepsilon_\text{i}$. At lower intensities photon interaction of the laser light can occur quantum-electro-dynamically with the electron in the hydrogen atom by resonance for energy levels given by energy eigen-values of the quantum states of the electron within the bound atom. At these lower intensities one cannot describe the electron interaction with the photons in the point-mechanical classical way as quiver motion. Nevertheless the correspondence to the quiver motion might be considered as a virtual oscillation (with all caution not to over-stress this description until it is studied further). It is therefore interesting to see when the product of the length of the quiver motion, $r$ of Eq. (8), with the momentum $p = mv$, Eq. (9) reaches the value of Planck's constant $h$

$$rmv = \hbar = h/2\pi. \tag{11}$$

This quantum relation permits substitution of the laser field $E$ of the quiver velocity $v$ in Eq. (9) with the quiver energy (10) to arrive at

$$\varepsilon_{\text{osc}} = \hbar^2/2mr^2. \tag{12}$$

For low laser intensities, this can be considered as a "virtual" quiver motion of the electron which is fundamentally different form the quantum states of the electron when bound in the atom. Based on this virtual description it is interesting to see what happens if $r$ assumes the value of the Bohr radius, Eq. (4). This arrives at a value of a "Bohr"-quiver oscillation energy

$$\varepsilon_{\text{B,osc}} = me^4/2\hbar^2 = \alpha^2 mc^2/2 = 13.6\ \text{eV}; \quad \alpha = e^2/\hbar c \tag{13}$$

using the vacuum speed of light c and the fine structure constant $\alpha$. This is just the ionization energy of hydrogen. It is worth noting that this value is simply expressed by the fine structure constant $\alpha$ and the electron rest energy $mc^2$. It is remarkable that this ionization energy for hydrogen marks a border line between the classical and the quantum mechanical state now expressed by the quiver motion, i.e. by the virtual electron oscillation.

The state of the inverted Rydberg cluster D(-1) [2] with distance $d^*$ of 2.3 pm between the deuterons (as measured from the 630 eV energy of the emitted deuterons with the TOF mass spectrometer) is a rather complicated quantum state for the electrons and deuterons. In fact it is the state of the electron of an unexcited hydrogen atom. Using the radius $r^* = d^*/2 = 1.15$ pm as a first approximation in the inverted Rydberg deuterium cluster in the same way as the Bohr radius (4) was used for hydrogen, we arrive at a virtual oscillation energy for the electron for "ionization" from the bound state in the inverter Rydberg state cluster.

$$\varepsilon_{\text{DD,osc}} = \varepsilon_{\text{B,osc}}(m_{\text{e}}/m_{\text{D}})(d/2r_{\text{B}})^2. \tag{14}$$

The laser intensity for producing a quiver energy $\varepsilon_{\text{B,osc}}$ for the boron ionization Eq. (14) at the wave length 565 pm is $2.298 \times 10^{14}$ W/cm$^2$ resulting with the same ratios as Eqs. (13) and (14) to arrive at a laser intensity

$$I^* = 3.1 \times 10^{10}\ \text{W/cm}^2. \tag{15}$$

This is close to the measured threshold in the experiment [23]. We assumed an ad hoc distance $d^*$ for the inverted D-clusters in these calculations in order to work with the radius $r^* = d^*/2$. The correct value may be somewhat different, leading to a slightly different threshold $I^*$. When very precise measurements of the threshold intensity are at hand, the more precise elongation of the virtual quivering in the D(-1) cluster could be calculated and may give more important information about the inverted Rydberg state of the D- which definitely will not be a "clumping" together of spheres but with much more complicated structures. This result and the corresponding theory also provide exciting new insights the correspondence principle of electromagnetic interactions [24].

It is very important to realize that the measured [2] $d^* = 2.3$ pm value is lower than the initially expected theoretical value. This shows qualitatively that the radius of an inverted deuterium atom is larger than for an inverted Rydberg state of free deuterium. This confirms the implied overlap between the cluster members with neighboring states within the D(-1) state under the assumption of a cubic deuteron lattice structure in the cluster. The degree of overlap can be defined quantitatively from this type of analysis and should be carefully studied when more precise measurements are obtained.

Compared to the very long time of the resonance transition of electrons in atoms, the virtual-quiver-model ionization process is indeed extremely fast, roughly in the femto-second range. The electric field amplitude of the laser of $10^{10}$ W/cm$^2$ is $2.7 \times 10^6$ V/cm in good consistence to a laser driven field emission process. The laser intensity threshold near $10^{10}$ W/cm$^2$ for removing the electrons in the inverted Rydberg state for the subsequent 630-eV Coulomb explosion of the deuterons arrives at the virtual electron oscillation energy in full analogy to the ionization of hydrogen. This represents a characteristic of the correspondence principle of electromagnetic interaction [24,29]. The inverted Rydberg state model may be considered at least as a heuristic. The essential same results are seen all in a direct Bose–Einstein cluster model.

## 6. Discussion and Conclusions

We underline the following open questions about comparison of 2 pm deuteron distance results for nuclear interaction. The first case of 2 pm was the semi-empirical fitting [9] of binary DD reactions using Coulomb screening by a factor 13 as confirmed in details by the complete quantum mechanical derivation by Czerski, Huke et al. [20,21]. The second case was the hypothetical model for explaining the Maruhn-Greiner local maximum of measured $P(Z)$ LENR heavy nuclear generation by assuming that the 2 pm screened distance motion of deuterons in the palladium crystal are "clumping" together to clusters with 150 deutrons of about 10 pm diameter to arrive at a Bose–Einstein state with nonlocality such that a compound nuclear reaction, Eq. (1), may produce the measured heavy nuclei. The third case is the inverted Rydberng state as a heusristic approach where there is a cubic lattice of Rydberg-inverted deuterium molecules in contrast to the BEC nonlocality.

The question may be whether there is some complementarity between BEC and inverted Rydberg states. This may be suggested from experimental results from the emission of nuclear reaction products using the volume clusters [1] for deuterons, where an increase of the fusion neutrons was measured when these states in targets for usual laser driven fusion were used [25,26]. Using surface state deuterium clusters, emission of MeV neutrons was measured at laser irradiation at about 20 times higher laser intensities $I$ above the threshold, Eq.(15) where the following gain of the MeV neutron numbers $N$ was increasing [27]

$$N \sim I^3. \tag{16}$$

This proportionality is comparable with the conclusion (see Fig. 1 of Ref. [28]) of the large scale laser fusion of spherically compressed thermal ignited direct drive volume ignition laser fusion with the proportionality

$$N \sim I^2. \tag{17}$$

This was confirmed [28] for ns laser pulses of about 10 kJ and is concluded for present experiments with up to 2 MJ pulses.

## References


[1] A. Lipson, B. J. Heuser, C. Castano, G. Miley, B. Lyakhov and A. Mitin, *Phys. Rev. B* **72** (2005) 212507.

[2] S. Badiei, P.L. Andersson and L. Holmlid, High-energy Coulomb explosion in ultra-high dense deuterium: Time-of-flight mass spectrometry with variable energy and flight length, *Int. J. Mass Spectr.* **282** (2009) 70–76.

[3] L. Holmlid, H. Hora, G.H. Miley and X. Yang, Ultra-high-denisty deuterium of Rydberg matter clusters for inertial confinement fusion targets, *Laser and Particle Beams* **27** (2009) 529–532.

[4] L. Holmlid, *Laser and Particle Beams* **31** (2013) 715–722.

[5] H. Hora and G.H. Miley, Maruhn–Greiner maximum from uranium fission for confirmation of low energy nuclear reactions LENR via a compound nucleuas with double magic numbers, *J. Fusion Energy* **26** (2007) 349–353, 357.

[6] George H. Miley, Heinrich Hora, Karl Philberth, Andrei Lipson and P.J. Shrestha, Radiochemical comparisons on low energy nuclear reactions and uranium, In *Low-Energy Nuclear Reactions and New Energy Technologies Source Book,* Vol. 2, Jan Marwan and Steven B. Krivit (Eds.), American Chemical Society/Oxford University Press, Washington DC, ISBN 978-0-8412-2454-4 (2009), pp. 235–252.

[7] Y.E. Kim and A.L. Zubarev, *Phys. Rev. A* **66** (2002) 053602.
[8] Yeong E. Kim, Theory of Bose–Einstein condensation mechanism for deuteron-induced nuclear reactions in micro/nano-scale metal grains and particles, *Naturwissenschaften* **96**(7) (2009) 803–811.
[9] H. Hora, J.C. Kelly, J.U. Patel, Mark A. Prelas, G.H. Miley and J.W. Tompkins, Screening in cold fusion derived from D–D reactions, *Phys. Let. A* **175** (1993) 138–143.
[10] M.A. Prelas, F. Boody, W. Gallaher, E. Leal-Quiros, D. Mencin and S. Taylor, *J. Fusion Energy* **9** (1990) 309.
[11] G.H. Miley and J.A. Patterson, *J. New Energy* **1** (1996) 11.
[12] G.H. Miley, G. Narne, M.J. Williams, J.A. Patterson, J. Nix, C. Cravens and H. Hora, *Progress in New Hydrogen Energy*, M. Okamoto (Ed.), New Energy and Industrial Technology, Tokyo, 1997, p. 629.
[13] H. Hora, Magic numbers and low energy nuclear transmutations by protons in host metals, *Czechosl. J. Phys.* **48** (1998) 321.
[14] H. Hora and G.H. Miley, *Czechosl. J. Phys.* **50** (2000) 433.
[15] J. Maruhn and W. Greiner, *Phys. Rev. Let.* **32** (1974) 548.
[16] J.A. Maruhn, W. Greiner and W. Scheid, Theory of fragmentation in fission, fusion and heavy ion scattering, in *Heavy Ion Collisions*, R. Bock (Ed.), Vol. II (North- Holland, Amsterdam, 1980), pp. 387–465.
[17] G.H. Miley, *Trans. Am. Nucl. Soc.* **76** (1997) 155.
[18] S. Ichimaru. Nuclear fusion in dense plasmas, *Rev. Mod. Phys.* **65** (1993) 255–299.
[19] K. Czerski, A. Huke and A. Biller et al., *Europhys. Lett.* **54** (2001) 449.
[20] A. Huke, K. Cerski and P. Heide et al., *Phys. Rev. C* **78** (2008) 015803.
[21] H. Hora, *Plasmas a High Temperature and Density*, Springer, Heidelberg, 1991.
[22] H. Hora and B.I. Henry, Polarization shift of spectral lines in high density plasmas, *Opt. Comm*. **44** (1983) 218–222.
[23] L. Holmlid, Private communication.
[24] Heinrich Hora and Peter H. Handel, Kapitza–Dirac effect with lasers and non-resonant interaction for quantum modulation of electron beams (Schwarz–Hora effect) , Appl. Phys. Lett. **102** (2013) 141119/1-4.
[25] X. Yang. G.H. Miley, K.A. Flippo and H. Hora, Energy enhancement for deuteron beam fast ignition of a pre-compressed inertial confinement fusion (ICF) target, *Phys. Plasmas* **18** (2011) 032703/1-5.
[26] Xiaoling Yang, George H. Miley, Kirk A. Flippo and Heinrich Hora. Hot spot heating process estimate using a laser-accelerated quasi-Maxwellian deuteron beam, *Laser and Particle Beams* **30** (2012) 31–38.
[27] Patrik U. Andersson and Leif Holmlid, Fusion generated fast particles by laser impact on ultra-dense deuterium: rapid variation with laser intensity, *J. Fusion Energy* **31** (2012) 249–256.
[28] H. Hora, Extraodrinary jump of increasing laser fusion gains experienced at volume ignition for combination with NIF experiments, *Laser and Particle Beams* **31** (2013) 228–232.
[29] B.W. Boreham and H. Hora, Energy spectra of electrons emitted from laser irradiated low density gas and the correspondence principle of electromagnetic interaction, *Laser and Particle Beams* **13** (1995)71–85.