



Research Article

4-Space Dirac Theory and LENR

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Abstract

A 4-space Dirac theory, which gives the same transition energies as the standard model, predicts specific distributions of virtual electrons and positrons accompanying bound electrons. If potential barriers are viewed stochastically, it seems possible that this has implications for LENR.

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1. Introduction

Those who seek a theoretical model for LENR face an immediate question. Does the explanation require an entirely new effect, lying outside currently accepted theory, or does it hinge on a hitherto overlooked consequence of the standard model? Given the complexities of the environments in which phenomena thought to be LENR are observed, the latter may seem more likely. Nevertheless, this paper presents an unconventional form of QED for the consideration of LENR researchers, in the hope that it may lead to theoretical progress.

Little-known versions of QED, known collectively as parametrized relativistic quantum theories [1,2], have for decades been investigated intermittently by researchers looking for models that clearly avoid any suggestion of a preferred frame of reference. In these models, the space-time coordinates $X^\lambda = (x^k, ct)$ are all on an equal footing, i.e. both spatial position and time are regarded as observables. An invariant parameter (called τ below), corresponding to the proper time of classical relativity, is used instead of t to describe the evolution of the wave function, which in the spin- $\frac{1}{2}$ theory proposed by the author [3] is a bispinor ψ satisfying a 4-space Dirac equation (Eq. (2)) that generalizes the conventional one.

As far as nonstandard QED is concerned, the first requirement is that its predictions must be extremely close – perhaps in many respects identical – to those of the standard model. For example, even a small departure from the energy levels of standard Dirac theory would immediately rule it out. Although the Dirac theory described below

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allows solutions in which the proper mass m_0 - an observable in the 4-space formulation - is not sharp, it also gives stationary solutions - eigenstates of both energy and proper mass - that are formally identical to those of conventional Dirac theory, and imply the same transition energies. However, the 4-space solutions (including those for a Coulomb potential) generally require a modified interpretation of their wave functions, because the 4-space picture includes contributions from both electrons and positrons, as follows.

The invariant $\psi^\dagger(i\gamma^4)\psi \equiv F(\mathbf{X}, \tau)$ is the expected space-time charge density, and the 4-vector $-\psi^\dagger\gamma^4\boldsymbol{\gamma}\psi \equiv \mathbf{J}(\mathbf{X}, \tau)$ is the expected particle current, so that $J^4 = \psi^\dagger\psi \geq 0$ implies flow in the positive time direction. Thus $F = F_1 - F_2$, where F_1 and F_2 are electron and positron densities, and $\mathbf{J}$ is the sum of the particle and antiparticle currents, which are assumed to have a common 4-velocity $\mathbf{U}$ given by $c\mathbf{J} = (F_1 + F_2)\mathbf{U}$. A second invariant, $Q \equiv \psi^\dagger\gamma^0\psi$, where $\gamma^0 \equiv -i\gamma^1\gamma^2\gamma^3$, is related to F and $\mathbf{J}$ by $\mathbf{J} \cdot \mathbf{J} = -(F^2 + Q^2)$. It follows [3] that

$$\begin{aligned} F_1 &= \left(\sqrt{F^2 + Q^2} + F \right) / 2, \\ F_2 &= \left(\sqrt{F^2 + Q^2} - F \right) / 2. \end{aligned} \quad (1)$$

The integrals of F_1 and F_2 over space-time do not generally represent whole numbers of charges, and we take this to mean that the expected 4-space densities may include contributions from virtual pairs, though usually at a very low level. We note that $F_2 = 0 \Leftrightarrow Q = 0$, and $F_1 = F_2 \Leftrightarrow F = 0$. We also find that F_1 and F_2 are interchanged by time reversal, as one might expect for electron and positron densities.

Later we use the Dirac representation, with its characteristic large and small components in a bound state: we then find that Q and (therefore) F_2 are normally small. However, in bound states Q increases with Z , giving rise to an increasing expected density of virtual pairs, and the model suggests pair creation when αZ is sufficiently close to 1. At the other extreme, a free particle has $Q = F_2 = 0$. Loosely speaking, Q measures the extent to which electromagnetic fields are producing virtual electrons and positrons - the effect increases with field strength. (Though as will be clear below, this does not mean that the electron and positron densities come separately from the large and small components of ψ .) Further details are given in [3,4], and the model has been illustrated by applying it to Klein's paradox [5].

2. Some General Results

The proposed 4-space Dirac equation is in general

$$\boldsymbol{\gamma} \cdot \left[\partial - \left(\frac{ie}{\hbar c} \right) \boldsymbol{\Omega} \right] \psi = \left(\frac{i}{c} \right) \frac{\partial \psi}{\partial \tau}, \quad (2)$$

where the 4-component spinor ψ is a function of the space-time coordinates $X^\lambda = (x^k, ct)$ and the invariant parameter τ . Some of the conventions used here are those of [3]; in particular, the Lorentz metric tensor is $\eta_{\alpha\beta} = \text{diag}(1, 1, 1, -1)$. The chiral representation was used in [5], but the usual Dirac representation is better suited to the present purpose. In [3], m was used for proper mass, but m_0 is employed here.

The 4-space theory has a proper mass operator $\hat{m}_0 = (-i\hbar/c^2)\partial/\partial\tau$, and imposing the condition for sharp proper mass, $\hat{m}_0\psi = m_0\psi$, we reduce (2) to the conventional form

$$\boldsymbol{\gamma} \cdot \left[\boldsymbol{\partial} - \left(\frac{ie}{\hbar c} \right) \boldsymbol{\Omega} \right] \psi = - \left(\frac{m_0 c}{\hbar} \right) \psi, \quad (3)$$

where $\psi = \mu(\mathbf{X})e^{im_0c^2\tau/\hbar}$.

In the 4-space formulation, energy–time uncertainty is put on the same basis as that relating momentum and position, and so a particle that is not in an eigenstate of energy has a non-uniform distribution in time. On the other hand, eigenstates of proper mass give distributions that are independent of τ . To be realistic, therefore, eigenstates of proper mass must also be eigenstates of energy – otherwise we get a non-uniform distribution in time that does not move forward in time as τ increases. We therefore now suppose that

$$\psi = v(\mathbf{x})e^{i(E_0\tau - Et)/\hbar}. \quad (4)$$

where v depends only on the spatial coordinates x^k , as indicated.

In the Dirac representation, if ψ is split into 2-component spinors ξ and η , it is found that the fundamental invariants defined above are

$$F = \xi^\dagger \xi - \eta^\dagger \eta; \quad Q = \xi^\dagger \eta + \eta^\dagger \xi. \quad (5)$$

Adopting the usual convention that ξ and η are respectively the large and small parts of ψ , we can write $\eta \sim \alpha Z \xi$. Provided Z is not too large, we shall therefore have $Q \ll F$, so that the electron and positron densities in Eq. (1) give

$$F_1 \approx F; \quad F_2 \approx Q^2/4F. \quad (6)$$

But (5) gives space–time densities, whereas we want the expected spatial density of charge. A self-consistent formulation requires [5] that in stationary states the spatial density is FU^4/c , where $\mathbf{U}$ is the 4-velocity of a spatial frame in which the state is stationary. Hence the expected spatial charge density of a stationary 4-space model in its preferred reference frame is just $F(x)$. However, this in general is different from the usual expected charge density, which is

$$J^4 = \psi^\dagger \psi = \xi^\dagger \xi + \eta^\dagger \eta, \quad (7)$$

and so a renormalization of ψ is needed: integrating over all space,

$$\int F(x) d^3x = 1. \quad (8)$$

3. The Ground-State Solution

While it is perhaps premature to devote much space to the details of specific states, it may be worth while to look at the ground state of a single electron bound to a nucleus of charge Z . We can omit the factor $e^{i(E_0\tau - Et)/\hbar}$ (see (4)), and specify the wave function as follows:

$$\begin{aligned} \xi_1 &= f(r), \\ \xi_2 &= 0, \\ \eta_1 &= \epsilon f(r) \cos \theta, \\ \eta_2 &= \epsilon f(r) \sin \theta e^{i\phi}, \end{aligned} \quad (9)$$

$$\epsilon = \frac{\beta}{1 + \gamma} = \frac{1 - \gamma}{\beta}, \quad (10)$$

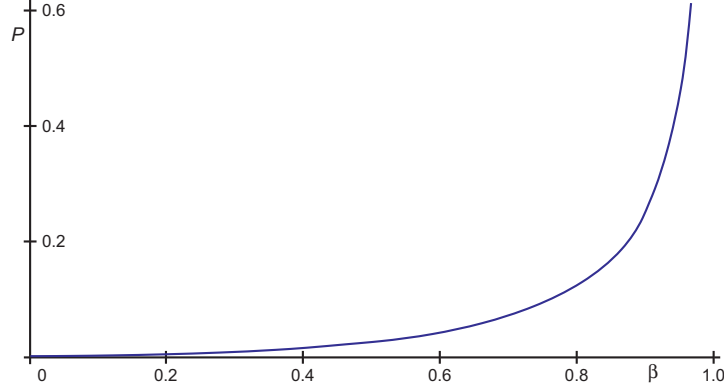


Figure 1. P , the space integral of the expected ground-state positron density, plotted against $\beta = \alpha Z$.

where $\beta = \alpha Z$ and $\gamma = \sqrt{1 - \beta^2}$. Here we have the usual ground-state radial distribution function $f(r)$, of the form

$$f(r) = K r^{\gamma-1} e^{-\lambda r}, \quad (11)$$

where K is a normalization constant and $\lambda = \beta m_0 c / \hbar$. See, for example, [6], p.79 (where $c = \hbar = 1$), or for more details, [7], p.69. The normalization (8) implies that

$$4\pi(1 - \epsilon^2) \int_0^\infty f^2(r) dr = 1. \quad (12)$$

Using the relations above, we find that the expected positron density (along with the balancing part of the electron density) is

$$F_2 = f^2(r) \frac{\sqrt{1 - \beta^2 \sin^2 \theta} - \gamma}{\gamma(1 + \gamma)}. \quad (13)$$

For small β this gives $F_2 \approx \frac{1}{4} \beta^2 f^2(r) \cos^2 \theta$, implying concentration around the polar axis. If we expand F_2 in powers of β , the second term is of order β^4 , and so the lowest approximation is a good one until Z becomes quite large.

For the ground state, using F (as in (5)) instead of J^4 (as in (7)) makes no difference to the expected net charge density, because in this case $\xi^\dagger \xi$ and $\eta^\dagger \eta$ differ only by a constant factor. Nevertheless, the ground state predicted by the present 4-space model is different from that of conventional first-quantized Dirac theory, in that it already accommodates positrons. Having normalized F , we can use the integral of the expected positron density, i.e. $P(\beta) = \int F_2 d^3x$ (over all space), to indicate the cumulative departure from standard predictions of charge distribution. The result (using $\gamma = \sqrt{1 - \beta^2}$, as above) is

$$P(\beta) = \frac{1}{4} \left[\frac{1}{\gamma} + \frac{\gamma}{2\beta} \ln \left(\frac{1 + \beta}{1 - \beta} \right) - 2 \right]. \quad (14)$$

When β is small, $P(\beta) \approx \beta^2/12$. Although $P(\beta)$ diverges as $\beta \uparrow 1$, it remains relatively small until β reaches about 0.6, as the the graph (Fig. 1) illustrates. Hydrogenic uranium (U^{91+}) has $\beta \approx 0.67$, $P \approx 0.06$.

By increasing Z , we eventually reach a singularity when $\beta \uparrow 1$. Although one knows that it must be preceded by pair creation in an intense electric field, this phenomenon is normally predicted only after second quantization. In the present

4-space model, however, we already have an indication of the physics. To reach $P = 1$ we need $\beta \approx 0.9837$, corresponding to $Z \approx 135$. In the context of LENR, the point of these remarks, and of the graph of $P(\beta)$, is that the predicted effects of the 4-space formulation are small in relatively weak electric fields, but can become significant in strong fields. A further illustration is provided by the 4-space treatment of Klein's paradox [5].

4. Discussion

The model presented here is an attempt to describe, within a first-quantized framework, phenomena that are normally in the domain of second quantization. In this sense it is complementary to the efforts of Barut and others [8–12] to enlarge the scope of first-quantized theories.

The virtual-particle distributions described above are distinct from the more familiar vacuum polarization. Both can be thought of in terms of quasi-classical charge distributions, but when we consider the implications for tunnelling, this picture may not be adequate. If we accept it, then the virtual electrons and positrons of the 4-space Dirac theory (as outlined above) simply cancel each other out. In reality, however, wave functions do not describe continuous distributions of charge, but rather expected values for the occurrence of discrete charges. The associated potentials will therefore exhibit random fluctuations, implying that (with a low probability) large deviations from the average must occur. This suggests that under suitable conditions there could be a significant breakdown of the usual Coulomb repulsion between hydrogen nuclei. But because such a breakdown is mediated by individual electrons, the likelihood of its occurrence should decrease very rapidly as nuclear charge increases, effectively restricting the phenomenon to hydrogen.

Stochastic barriers have already been considered by Habib [13] and others. In the stochastic picture, low-probability extremes in barrier fluctuations are crucial, so that the fine detail of charge distributions takes on added significance. Concentration of virtual particles and antiparticles around an axis will make the extremes more likely, and thus provide a preferred route through a potential barrier. The foregoing description of the ground state shows that such effects can be much greater in the presence of strong electric fields. Even if the details outlined above do not apply in an actual LENR experiment, the breaking of symmetry (from spherical to axial in the ground state) can be expected to hold quite generally. In a lattice structure, especially, it may provide pathways along which low-energy hydrogen nuclei are more likely to have their mutual repulsion nullified by intervening electrons.

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