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Research Article

Theory of Fusion During Acoustic Cavitation in C_3D_6O Liquid

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Abstract

This paper demonstrates that an exact calculation for transition probability of deuteron under the phonon–deuteron interaction leads to violation of Fermi golden rule. Considering the violation of Fermi golden rule, the zero-point oscillation, and the energy uncertainty relation, this paper demonstrates that the neutron emission during acoustic cavitation comes from D(entered)–D(entered) fusion in C_3D_6O liquid instead of in cavitation vapor bubbles. This paper gives some predictions. The most important prediction is that the water can be taken as energy source producing fusion instead of C_3D_6O liquid.

Keywords: Acoustic cavitation, Fermi golden rule, Fusion, Green function, Phonon–deuteron interaction, Transition probability

1. Introduction

Taleyarkhan et al. measured time spectra of neutron in acoustic cavitation experiments with chilled deuterated acetone [1–3]. Statistically significant neutron emission was measured. The neutron emission rate was up to $\sim 4 \times 10^5$ n/s, where n indicates neutron. Taleyarkhan et al. thought that the neutron emission is due to D–D fusion inside the cavitation vapor bubbles nucleated in highly tensioned deuterated acetone by means of fast neutrons. Here, D indicates deuteron. The D–D fusion mechanism imagined by Taleyarkhan et al. is as follows. The intense implosive collapse of bubbles, including acoustic cavitation bubbles, can lead to extremely high compression, temperature (about 10^9 K), and density of D. Their duration is large enough. These conditions yield the D–D fusion inside the cavitation vapor bubbles.

The difficulty of Taleyarkhan et al.'s mechanism of D–D fusion is that there is no direct experimental evidence to show the high temperature of 10^9 K in the cavitation vapor bubbles. As far as we know, there is still no exact microscopic theory for the D–D fusion during acoustic cavitation. Basing on the exact calculation of the transition

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probability of deuteron under phonon–deuteron interaction, this paper proposes a microscopic theory for the D–D fusion. Our mechanism is as follows. During the acoustic cavitation a large number of D inside the bubbles enter the spherical surface layer of the chilled deuterated acetone by means of high compressions and high density of D. Note that the spherical surface layer surrounds the cavitation vapor babbles. We call the D entered the spherical surface layer the entered D, and is expressed as D_e or D(entered). The state of D_e in the spherical surface layer is of non-equilibrium and of high density. We call the spherical surface layer D_e–C_3D_6O system. There is D_e–phonon interaction in the spherical surface layer. We will demonstrate that the D_e–phonon interaction leads to the D_e–D_e fusion in the spherical surface layer. In view of the importance of exact calculation of the transition probability to this paper, we introduce two another examples in Section 2, taken from Ref. [4]. In Section 3, we demonstrate that the transition probability of D_e under D_e–phonon interaction violates Fermi golden rule, and give a microscopic theory for the D_e–D_e fusion in the spherical surface layer. In Section 4, we make some predictions.

2. Departure from Fermi Golden Rule

Liu et al. pointed out that it has to be emphasized that two assumptions were made in calculating the transition probability previous to Ref. [4]. Suppose the Hamiltonian H can be put in the form: $H = H^{(0)} + V$, where $V = A\exp(\mathrm{i}\omega t) + A^{+}\exp(-\mathrm{i}\omega t)$, $|a\rangle$ is a discrete state of $H^{(0)}$, $|b\rangle$ is the state in a continuous spectrum of $H^{(0)}$, B is the domain of $|b\rangle$, A is a time-independent operator, and that at initial time t_0, which is taken to be zero, the system is in the state $|a\rangle$. The probability $P_{a\to B}$ of transition into one of the states in the domain B at time t by absorbing an energy $\hbar\omega$ has been given in Ref. [4].

$$P_{a\to B} = \int_B P_{a\to b}\rho_b(E_b)\,\mathrm{d}E_b, \tag{1}$$

$$P_{a\to B} = \frac{1}{\hbar^2}|A_{ba}|^2 f(t, \omega_b - \omega_a - \omega), \tag{2}$$

$$f(t, \omega_b - \omega_a - \omega) = \frac{\sin^2[(\omega_b - \omega_a - \omega)t/2]}{[(\omega_b - \omega_a - \omega)/2]^2}, \tag{3}$$

$$\lim_{t\to\infty} f(t, \omega_b - \omega_a - \omega) = 2\pi t\delta(\omega_b - \omega_a - \omega), \tag{4}$$

where $\rho_b(E_b)$ is the density of states at E_b, $E_b = \hbar\omega_b$, $E_a = \hbar\omega_a$, and $A_{ba} = \langle b|A|a\rangle$. Taking $-\epsilon/2 < E_b < \epsilon/2$. The first assumption in calculating the integral in Eq. (1) is that the width ϵ is so sufficiently small that A_{ba} and ρ_b remain practically constant over the integral so that they can be taken outside the integral sign in Eq. (1). The second assumption is that t is sufficiently large for ϵ to be much greater than the period of oscillation of the f function in Eq. (3), i. e., $\epsilon \gg 2\pi\hbar/t$, so that Eq. (4) can apply. Under the two assumptions $P_{a\to B}$ is then [4]

$$P_{a\to B} = \frac{2\pi}{\hbar}|A_{ba}(E_b)|^2\rho_b(E_b)t. \tag{5}$$

The $P_{a\to B} \propto t$ may then be called the Fermi golden rule. Ref. [4] demonstrated that the exact calculations of transition probability, without using the two assumptions, lead to violation of Fermi golden rule in the cases of hydrogen ionization and KWW relaxation.

3. Departure from Fermi Golden Rule and Theory of D_e–D_e Fusion

The intense implosive collapse of cavitation vapor bubbles can lead to extremely high compressions and high density of D [3]. During this process, a large number of D inside the bubbles enter the spherical surface layer, surrounding the cavitation vapor bubble, of the chilled deuterated acetone, and form a D_e–C_3D_6O system. The C_3D_6O is a liquid. The model of translational motion in liquid, proposed by Kruus [5] and Larsson [6], is as follows. Assume a molecule to undergo vibration in its cell on the average for a time τ_0. It then is displaced into another cell through some distance l, where it again oscillates for a time τ_0, etc. We can describe such collective and translational motion by longitudinal phonon [5,6]. The density oscillations can also be described by the longitudinal phonons [5,6]. Neutron scattering data for liquid had been interpreted in terms of the presence of longitudinal phonon successfully [5,6].

For the D_e–C_3D_6O system the explicit form of the D_e-longitudinal phonon interaction Hamiltonian, $H_{\rm int}$, in the deformation potential approximation is [7, 8]

$$H_{\rm int} = \sum_{m,p} c_{\rm m}^{+} a(\mathbf{p}) c_{\rm m} C i \left[\frac{\hbar}{2NM\omega}\right]^{1/2} \mathbf{p}\cdot\mathbf{e}(\mathbf{p}) + h.c., \tag{6}$$

where N is number of C_3D_6O molecules in the D_e–C_3D_6O system, $c_{\rm m}$ the annihilation operator of D_e at site m, $a(\mathbf{p})$ the phonon annihilation operator, M the mass of C_3D_6O molecule, $\mathbf{e}(\mathbf{p})$ the longitudinal polarization unit vector, $\mathbf{p}\cdot\mathbf{e}(\mathbf{p}) = p$ for longitudinal phonon, p is the wave number of phonon, ω the phonon frequency, C the deformation potential coefficient. Let us estimate the value of C in C_3D_6O liquid. The density of C_3D_6O liquid is 0.79 g/cm^3 [9]. The average volume occupied by one C_3D_6O in C_3D_6O liquid is thus (4.93 Å)^3. The average volume occupied by one atom in a C_3D_6O molecule of C_3D_6O liquid is thus $\Omega = (2.28\ \text{Å})^3$. The average charge of every atomic nucleus in C_3D_6O molecule is -1.8 e, where e is the free electron charge. For C_3D_6O liquid, the average interaction between D_e and atomic nucleus of C_3D_6O molecule is $\epsilon = 1.8e^2/2.28$ Å. From Ref. [8], $\delta\epsilon = C\delta(N_1\Omega)/(N_1\Omega)$, where N_1 is the number of atoms in C_3D_6O liquid. Thus, we obtain $C = -6.03\times10^{-12}$ erg.

Let us assume that the D_e is at m at $t = 0$. The probability amplitude of finding the D_e still at m at $t > 0$ is the following Green function of time at a finite temperature [7]

$$G(t) = -i\langle T c_{\rm m}(t) c_{\rm m}^{+}(0)\rangle. \tag{7}$$

A waiting-time distribution $Q(t)$ is $Q(t) = |G(t)|^2$ [10]. The lifetime of D_e at m, $\langle t\rangle$, is given as [10]

$$\langle t\rangle = -\int_0^\infty t\ \mathrm{d}Q(t). \tag{8}$$

From an exact derivation, Ref. [7] gives

$$Q(t) = \mathrm{e}^{-P(t)}. \tag{9}$$

$P(t)$ is then expressed as [7]

$$P(t) = \frac{2C^2}{\hbar NM}\sum_p [2n(\mathbf{p})+1]\frac{p^2}{\omega}\frac{\sin^2(\omega t/2)}{\omega^2}, \tag{10}$$

where $n(\mathbf{p})$ is the Bose distribution of the phonon. We will now just consider the zero-point energy term in Eq. (10). Because the signs of the two terms in Eq. (10) are the same the $P(t)$ will become larger if we consider also the term with the Bose distribution of the phonon. For the zero-point energy term, Eq. (10) can be transformed into the integration

$$P(t) = \frac{2C^2}{\hbar NM} \int_0^{\omega_{up}} \mathrm{d}\omega \rho(\omega) \frac{p^2}{\omega} \frac{\sin^2(\omega t/2)}{\omega^2}, \tag{11}$$

where $\rho(\omega)$ corresponds to the density of states of longitudinal phonon, and ω_{up} is the upper limit of spectral density function. Here, we meet the same function as Eq. (1). We do not use the two assumptions mentioned in Section 2, and make an exact calculations for Eq. (11). We know from Eq. (4.6) of Ref. [11] that for a completely random lattice the p^2 in Eq. (11) represents $|\mathbf{k} - \mathbf{k}'|^2$, where $\mathbf{k}$ and $\mathbf{k}'$ are the wave vectors of a particle before and after absorption of a phonon. The $|\mathbf{k} - \mathbf{k}'|^2$ does not depend on the energy for a completely random lattice [11]. The average value of $|\mathbf{k} - \mathbf{k}'|$ in the Brillouin zone, $|\mathbf{k} - \mathbf{k}'| = \bar{p} = \pi/(2a)$, is taken for a completely random lattice [11]. For a perfect lattice $|\mathbf{k} - \mathbf{k}'| = p = \omega/v$ where v is the longitudinal sound velocity [11]. However, considering that the phonons in liquid have a much-shorter lifetime and pathlength than in crystalline solids [5] and that liquids have short-range order ($\sim$ 20 Å) [5], one often takes $p = \omega/v$ in liquid theory approximately [5]. Actually, $p = \omega/v$ is correct only for a perfect lattice. Actual solid and liquid have more or less deviations from the perfect lattice. Especially, the long-range disorder in liquid still has a little influence to the relation $p = \omega/v$. Therefore, we should imitate Refs. [4, 11] to take

$$p = \left(\frac{\omega}{v}\right)^{1-\beta'} \bar{p}^{\beta'}, \tag{12}$$

where $\bar{p} = \pi/(2 \times 4.93$ Å), and $\beta' \approx 0$ for C_3D_6O liquid. To obtain the express of $\rho(\omega)$ in Eq. (11), we make the following analyses. We will show that the D_e–D_e fusion is induced by the D_e-longitudinal phonon interaction in C_3–D_6O liquid. To have this interaction the D_e in the C_3D_6O liquid should be in non-equilibrium state, and thus can move. In this movement, the D_e can absorb longitudinal phonon. If the D_e is trapped in some place of the C_3D_6O liquid, and thus moves together with the molecule C_3D_6O, then the movement of the D_e becomes a part of the oscillation of molecule C_3D_6O. In this case there is no D_e–phonon interaction. We assume for specification that the non-equilibrium state of D_e just occurs in the spherical surface layer whose thickness is only one C_3D_6O molecule. For this quasi-two-dimensional D_e–C_3D_6O system, the average density of states of phonon, $\rho(\omega)$, is $10^{13}S/(4\pi v^2)$ [12]. The S is the total area of the spherical surface layer of the D_e–C_3D_6O system. The v is the longitudinal sound velocity in C_3D_6O liquid. $v = 1174$ m/s at 25°C [9]. If we set $\beta = 1 - 2\beta'$, then $\beta \approx 1$. Let $A = C^2\pi^{2\beta'}S/[\pi\hbar NMv^{4-2\beta'}(2 \times 4.93\ \text{Å})^{2\beta'}]$, then Eq. (11) becomes

$$p(t) = A \int_0^{\omega_{up}t} \mathrm{d}(\omega t) \frac{\sin^2(\omega t/2)}{(\omega t)^{2-\beta}} t^{1-\beta}. \tag{13}$$

Now let us assume that at $t = 0$ we have a D_e at site m with energy E_m and a phonon with frequency ω_{up}, and at $t > 0$ we have only a D_e at the site m. For this physical system, the uncertainty relation for energy, Eq. (44.1) of Ref. [13], gives $\omega_{up}t = 1$ for any value of t. Assume $\beta = 0.8$. Numerical calculation for the integral in Eq. (13) gives 0.1335. Eq. (13) then becomes

$$P(t) = 0.1335At^{0.2}. \tag{14}$$

From Eq. (14) we see that the Fermi golden rule is violated seriously. However, we should believe this result because it comes from exact calculations for Eq. (11) without using the two assumptions in Section 2.

Set

$$\tau = (0.1335\ A)^{-1/0.2}, \tag{15}$$

then

$$P(t) = \left(\frac{t}{\tau}\right)^{0.2}. \tag{16}$$

Substituting Eq. (16) into Eqs. (9) and (8) gives that the lifetime of D_e at m under the influence of D_e-longitudinal phonon interaction is

$$\langle t\rangle = \tau\Gamma\left(1+\frac{1}{0.2}\right), \tag{17}$$

where Γ indicates Gamma function. Numerical calculations give $\langle t\rangle = 3.8\times10^{-26}$ s. Considering that the D_e with radius 10^{-13} cm has possibility to move to all directions, the probability of D_e to move to a specific D_e is $P = \pi(10^{-13})^2/4\pi a^2$, where a is the nearest distance between two moving D_es in the spherical surface layer of C_3D_6O liquid.

If we take $a = 2$ Å, then $P = 6.25\times10^{-12}$. The D_e in the D_e–C_3D_6O system makes random thermal movement. At 300 K, the velocity of D_e is thus $v_D = 1.1\times10^5$ cm/s. If we do not consider that the D_e has possibility to move to all directions, then the collision rate of a D_e with another D_e is $v_D/2$ Å. If just before the wink of collision between two D_es one D_e absorbs a phonon, then its lifetime is 3.8×10^{-26} s, and the energy given by the energy uncertainty relation is 2.76×10^{-2} erg. Noting that even at 10^9 K the energy of D_e is only 1.38×10^{-9} erg, we can say that the fusion of the two D_es can occur definitely. (If $\beta = 0.9$, then the lifetime is 2.28×10^{-36} s, and the energy given by the energy uncertainty relation is 5×10^8 erg.) In our numerical calculations the estimative value of C is rather rough because it is based on the average volume and the average charge. The larger the value of C, the shorter the lifetime. If $C = -3\times10^{-12}$ erg instead of -6×10^{-12} erg, then the lifetime is 4.0×10^{-23} s instead of 3.8×10^{-26} s. In this case the energy given by the energy uncertainty relation is 2.76×10^{-5} erg, which is still much larger than 10^9 K. The nearest distance between two D_es is 2 Å. The number of D_es in this length of 2 Å is, in principle, $2\times10^{-8}/10^{-13} = 2\times10^5$. If the D_e absorbs only one phonon in the length of 2 Å, then the probability to absorb a phonon by D_e just before collision wink is only $P' = 5\times10^{-6}$. Considering all the factors, the rate that a moving D_e comes into collision with another D_e and produces D_e–D_e fusion is $R = P'Pv_D/2\ \text{Å} = 3.3\times10^{-4}$/s. If the number of cavitation vapor bubbles in the chilled deuterated acetone is 100 and the radius of the bubbles is 120 nm in average, then the total volume of the spherical surface layer of the 100 vapor bubbles is 9.7×10^{-15} cm^3. Noting that the nearest distance between two moving D_es is 2 Å, then the total number of moving D_e in this volume is $N_t \approx 9.7\times10^{-15}/(2\times10^{-8})^3 = 1.2\times10^9$. The total fusion rate is $RN_t = 4\times10^5$/s. This is just the observed value in Ref. [3]. The observed radius of the cavitation bubble is from tens of nanometers to several hundreds of nanometers [3]. If we take the nearest distance between two moving D_es $a = 3$ Å instead of 2 Å, then the total fusion rate is 0.8×10^5/s, which has no change of order of magnitude in comparison with 4×10^5/s. Therefore, we can say that, considering the energy uncertainty relation, the violation of Fermi golden rule, and the zero-point oscillation in the D_e–C_3D_6O system, the experimental results in Refs. [1–3] can be explained by D_e–D_e fusion on the whole.

4. Predictions

Our theory supports the D_e–D_e fusion in Refs. [1–3] and predicts that there should be excess heat. From ${}^2_1\mathrm{D} + {}^2_1\mathrm{D} \longrightarrow {}^3_2\mathrm{He} + {}^1_0\mathrm{n} + 3.3$ MeV and the observed neutron emission rate ($\approx 4 \times 10^5$ n/s), we know the excess heat rate is $\approx 1.3 \times 10^6$ MeV/s. The reason that the excess heat is difficult to be observed in Ref. [3] is that the volume of the total spherical surface layer is too small (only 9.7×10^{-15} cm^3), and thus the number of moving D_es are too small. The fusion imagined by Refs. [1–3] occurs in the cavitation bubbles. However, our theory demonstrates that the fusion occurs in liquid.

In our theory for the D_e–D_e fusion during acoustic cavitation, the sufficient conditions for occurrence of the D_e–D_e fusion are the existences of D_es in high density non-equilibrium state and the longitudinal phonons in a condensed matter system. If the compression and the density of moving H in the acoustic cavitation vapor bubbles in water are high enough, then a number of moving H can enter the spherical surface layer surrounding the cavitation bubbles and form non-equilibrium state of the entered H, at least, in a layer with thickness of one molecule of water. The H(entered)-H(entered) fusion can also occur in the spherical surface layer outside the bubbles, i. e., in water. However, there was no statistically significant neutron emission, when one used C_3H_6O liquid instead of C_3D_6O liquid [3]. We think that the H(entered)-H(entered) fusion already occurred in Ref. [3], when one used C_3H_6O liquid. The reason why is simple. The last products of H(entered)-H(entered) fusion are ${}^4_2\mathrm{He}$ and excess heat. Therefore, that there is no neutron emission does not imply that there is no H(entered)-H(entered) fusion in C_3H_6O liquid. If the H(entered)-H(entered) fusion in water can be realized in suitable acoustic cavitation conditions, then the water can provide nearly infinite energy source for mankind. The cavitation vapor bubbles in water can be nucleated by fast neutron or by laser.

Considering that the energy given by the energy uncertainty relation is very large (2.76×10^{-2} erg), the moving D_e (or H(entered)) can overcome the high potential barriers between the moving D_e (or H(entered)) and any atom in C_3D_6O (or C_3H_6O and H_2O). Therefore, we predict that there are fusions between D_e (or H(entered)) and any atom of C_3D_6O (or C_3H_6O and H_2O) as well.

After the finding of neutron emission during acoustic cavitation in Refs. [1–3], most people doubt the correctness of quantum mechanics. Our theory clearly shows that the fusion in Refs. [1–3] is true, and the quantum mechanics is correct. Although the exact calculations in this paper and in Ref. [4] show that the Fermi golden rule is broken in many cases, yet the Fermi golden rule is not the foundation of quantum mechanics. Our theory just shows that when one uses the Fermi golden rule, one should be careful enough.

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