J. Condensed Matter Nucl. Sci. 39 (2025) 1–13

Research Article

Relativistic Motion of the Electron and the Heaviest Element

C. A. M. dos Santos

Escola de Engenharia de Lorena, Universidade de São Paulo, Lorena-SP, 12.602-810, Brazil

M. S. da Luz

Instituto de Ciências Tecnológicas e Exatas, Universidade Federal do Triângulo Mineiro, Uberaba-MG, 38.025-180, Brazil

F. S. Oliveira

Instituto de Física "Gleb Wataghin", Universidade Estadual de Campinas, Campinas-SP, 13.083-970, Brazil

O. Rousselle

E. D. Physique en Ile de France, Université PSL, Sorbonne Université, Université Paris-Saclay, 75005 Paris, France

Abstract

The relativistic motion of the electron in the toroidal model is reported. The different motions of the center of mass and the charge provide a direct way to describe the relativistic effect on the helical motion of the electron. The momentum-energy relation is deduced in easy manner, which led to a simple interpretation of the electron energy. The relativistic effect on the helical motion due to the ratio of the Compton and the Bohr radii in the Bohr atom with Z protons is compared with the nucleus radius, providing a way to estimate the heaviest element of the periodic table, which is found to be between $Z = 122$ and 137.

Keywords: *Zitterbewegung*; Theory of Relativity; The Heaviest Element

1. Introduction

During the last century many authors have discussed how to properly incorporate relativistic aspects to the motion of the electron and other quantum particles. Paul Dirac was the firstly to introduce the relativistic version of the Schrödinger equation in 1930's [1]. One of the important results of the theoretical prediction is the *zitterbewegung* motion of the electron [2], which is still under intense debate whether is a real physical effect or a mathematical artifact of the Dirac equation [3]–[5].

In a completely different way, authors working on the theory of relativity have argued that a rotating charge has a gyromagnetic ratio $g = 2$ and dimensions related to the Compton wave length, suggesting that electron has structure which is related to theory of relativity [6], [7]. This is still a subject of study in order to unify quantum mechanics and general relativity [8], [9].

Thus, one of the most important questions still open today is whether or not the electron and other quantum particles have structure. In quantum field theory, electron is considered a point-like particle described by a wave function, accounting for its wave-particle duality [10], [11]. On the other hand, many authors have considered that the electron has structure, interpreting the *zitterbewegung* effect as evidence that electron travels at the speed of light in a circumference of Compton radius (reduced Compton wave length, $r_C = \lambda_C/2\pi$) [12]–[32].

Regarding the latter group [33], there are two current ideas regarding the electron structure and the *zitterbewegung* effect, one in which the electron is considered as massless charge traveling at the speed of light [14], [20], [22] and another in which the *zitterbewegung* is related to the different motions of the electron center of mass and the charge [15], [19], [27]–[29].

In favor of the separated motion of the electron center of mass and the charge our previous works have shown that: *i*) the solution of the Maxwell equations using the so-called Schwinger electromagnetic wave naturally leads to the correct values for the gyromagnetic ratio, the orbital current, and the charge of the electron, also suggesting that the center of mass has a different motion than the charge [27]; *ii*) the prediction of the Bohr postulates as a consequence of the helical motion of the electron charge [28]; and *iii*) the centripetal force acting on the electron mass and charge in different positions, which show different oscillations with same *zitterbewegung* frequency [30].

The theoretical prediction by Rivas [15], [19] regarding the separated motion of the electron center of mass and center of charge is in excellent agreement with the electron toroidal model, in which the charge orbits the mass with a circumference of the size of Compton radius, traveling at the speed of light. This issue will be explored in more detail in this report.

Although this paper has some similarities with other recent works [20], [30], [32], [34]–[36], it describes a simple approach to understand the relativistic effect on the electron motion in the toroidal model taking into account the separated motion of the center of mass and the center of charge. The relativistic energy-momentum relation is also naturally deduced as a consequence. Furthermore, the relativistic changes in helical motion of the electron in the first orbit of atoms with Z protons provide a new way to estimate which is the heaviest element of the periodic table.

2. Results and Discussion

2.1. Relativistic Motion of the Electron

In this section we describe the relativistic effect on an object with mass m traveling with a velocity v along with an electromagnetic wave, which runs along the transverse direction to the motion in a circumference of radius r' in the rest frame.

Imagine an astronaut inside a rocket observing a ring made out of optical fiber with refraction index equal to 1 or a tube with perfect reflection inside, travelling along with him (rest S' frame) at a relativistic velocity v.

If an electromagnetic wave is inserted on the top of the ring, the astronaut travelling along with it will see the wave to expend a period T' to make one turn of $2\pi r'$ around the ring, no matter the velocity v (this is similar to a clock made out of electromagnetic wave).

Another observer at the rest with regard to the rocket (boosted S frame) will see a different motion of the electromagnetic wave, which will look like a spiral. Figure 1 displays the different ways the observers in S and S' frames see the motion of the light.

Defining the period at the rest (S') and boosted (S) frames as T' and T, respectively, for an electromagnetic wave making one turn in the blue line shown in Figure 1 (a) and (b), special relativity predicts a time dilation between both

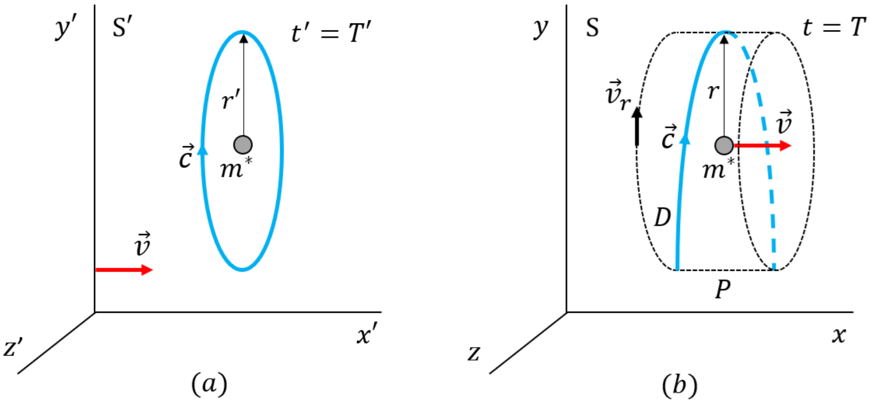

Figure 1. In (a) is shown what an observer at the rest frame (S') sees at instant $t' = T'$, when electromagnetic wave makes one turn in the circumference $2\pi r'$ (this is a little different than the Figures 6(i) and 6(ii) by Natarajan [36] since no wave turn is observed at $t =$ zero). (b) An observer in the boosted frame (S) sees the electromagnetic wave traveling in a helical way with distance D and a pitch P. A complete turn is observed by this observer at $t = T$. In both frames $r = r'$ and m^* is the equivalent mass of the electron, which will be defined later on in the main text.

frames in the way that

$$T' = \frac{2\pi r'}{c} \tag{1}$$

and

$$T = \frac{D}{c}, \tag{2}$$

providing

$$T = \frac{1}{\sqrt{1 - v^2/c^2}} T', \tag{3}$$

which is the same time dilation predicted for the "classical" (linear wave) special relativity theory [10], [37].

Furthermore, the velocity along the circumference (v_r) at the boosted frame shown in Figure 1(b), is given by

$$v_r \equiv \frac{2\pi r}{T} = \sqrt{1 - v^2/c^2} \frac{2\pi r'}{T'}, \tag{4}$$

where

$$\frac{2\pi r'}{T'} = c, \tag{5}$$

leading to

$$v_r = c\sqrt{1 - v^2/c^2}, \tag{6}$$

or

$$c^2 = v_r^2 + v^2. \tag{7}$$

Furthermore, the rotating frequency in S and S' frames are obtained by inversing Equation (3), *i. e.*

$$\nu' = \sqrt{1 - v^2/c^2}\nu. \tag{8}$$

which is in agreement with the discussion by de Broglie [38].

Based on geometrical arguments, one can easily show that

$$D = \sqrt{4\pi^2 r^2 + P^2}, \tag{9}$$

where $P = vT$ and $r = r'$.

In addition, in a period T, the relevant distances traveled during this time seen by the observer in the boosted frame can be calculated by

$$T = \frac{2\pi r}{v_r} = \frac{P}{v} = \frac{D}{c}, \tag{10}$$

which imply

$$D = \frac{2\pi r}{\sqrt{1 - v^2/c^2}}, \tag{11}$$

and

$$P = \frac{v}{c}\frac{2\pi r}{\sqrt{1 - v^2/c^2}}, \tag{12}$$

after using Equation (6).

Furthermore, Equation (7) leads to a direct way to find the Einstein energy-momentum relation [10], [37]. Multiplying both sides of the Equation (7) by m^2c^2, one can find

$$m^2c^4 = m^2c^2\left(c\sqrt{1 - (v/c)^2}\right)^2 + m^2v^2c^2, \tag{13}$$

which after taking into account the relativistic mass given by

$$m = \frac{m_0}{\sqrt{1 - (v/c)^2}} = \gamma m_0, \tag{14}$$

where m_0 is the rest mass and $\gamma = 1/\sqrt{1 - (v/c)^2}$ is the Lorentz factor, along with linear momentum ($p = mv$) and the relativistic energy ($E = mc^2$), the energy-momentum relation is naturally found as

$$E^2 = p^2c^2 + m_0^2c^4 \tag{15}$$

or

$$E = \frac{m_0c^2}{\sqrt{1 - (v/c)^2}} = \gamma m_0 c^2, \tag{16}$$

which has been found in other recent works [30], [39]. Thus, Equations (11) and (12) show the importance of helical motion of the electromagnetic wave to understand the relativistic effects of quantum particles in a simple way.

In order to better understand this behavior, we explore some cases reported on the helical motion of the electron, which has been recognized as related to the *zitterbewegung* behavior [33].

Taking Equation (11) as a reference for the helical motion, there are four cases to be considered: *i*) two cases related to the increase of the D due to the time dilation taking constant r; and *ii*) other two, which assume that the length D is invariant to the motion. All cases are discussed as follows.

In special relativity, a Lorentz boost does not affect the transversal part of an object. So, the radius of the ring shown in Figure 1 is the same for the different observers in S and S' frames, *i. e.*, $r = r' =$ constant, which has two cases reported on literature related to the

$$r = r' = \frac{\lambda_C}{2\pi}, \tag{17}$$

where the circumference has to do with reduced Compton radius (r_C) [20], [21], and

$$r = r' = \frac{\lambda_C}{4\pi}, \tag{18}$$

in which the circumference is due to half of the reduced Compton radius, as highlighted in [13] and [30], since it provides the correct values for the spin ($\hbar/2$) and *zitterbewegung* frequency ($2mc^2/h$) [13], [14], [24], [30]. In these two cases, D and P increase as v increases.

The others two cases suppose that the radius of the ring (r and r') in Figure 1 varies with the velocity v. In such cases, Equations (9) to (12) still the same. One of the most important possibilities is to establish that [40]

$$r = r' = r_C\sqrt{1-(v/c)^2} = \frac{\lambda_C}{2\pi}\sqrt{1-(v/c)^2}. \tag{19}$$

Taking this case, D and P become

$$D = 2\pi r_C = \lambda_C, \tag{20}$$

which is v independent, and

$$P = \frac{v}{c}2\pi r_C = \frac{v}{c}\lambda_C. \tag{21}$$

An interesting fact is that Equation (21) agrees with the Bohr postulates [28]. The Bohr velocity for the first orbit in the hydrogen atom is given by $v = v_B = \alpha c$, which provides

$$P = \alpha\lambda_C = \lambda_e, \tag{22}$$

where λ_e is related to the classical electron radius ($r_e = \lambda_e/2\pi$) [27], [41].

Furthermore, the velocity of the n-th orbit in the Bohr atom can be calculated by

$$v_n = \frac{P_n}{T} = \frac{\lambda_e/n}{\lambda_C/c} = \frac{\alpha c}{n} = \frac{v_B}{n}, \tag{23}$$

where P_n is the pitch for the n-th Bohr orbit. Equations (22) and (23) provide the natural quantization of the orbits in the Bohr atom, as demonstrated recently by some of us based on the helical motion of the electron [28].

Similar results to P and v_n are obtained for the two first cases using the limit $v \ll c$ ($v_B/c = \alpha$) in the Equations (11) and (12), keeping $r = r'$ = constant.

Now, taking $\lambda = 2\pi r = 2\pi r' = \lambda'$ and using Equation (19), the radius dependence of the velocity also implies that

$$\lambda = \lambda' = \lambda_C\sqrt{1-(v/c)^2}, \tag{24}$$

demonstrating that the electromagnetic wave energy (photon energy) in S' frame can be calculated by

$$E' = \frac{hc}{\lambda'} = \frac{hc}{\lambda} = \frac{hc}{\lambda_C\sqrt{1-(v/c)^2}} = \gamma\frac{hc}{\lambda_C}, \tag{25}$$

which is the same as the mass energy, since

$$E' = mc^2 = \frac{m_0c^2}{\sqrt{1-(v/c)^2}} = \gamma m_0 c^2, \tag{26}$$

after remember that $\lambda_C = h/m_0c$.

In addition, Equations (16), (25), and (26) lead to $E = E'$, showing that the energy is frame invariant in the present case, which also implies

$$\lambda_C m_0 = \lambda m = \frac{h}{c}, \tag{27}$$

that is frame invariant too and defines the Compton wavelength (λ) at any velocity v. It is interesting that Equation (27) is reminiscent of a type of the angular momentum conservation, which is similar to one of the Bohr postulates [10], [28].

In fact, these observations suggest that the helical motion of the electron due to the *zitterbewegung* behavior, is a fundamental characteristic, which is important for describing the physical properties of quantum particles, extensively reported by many authors nowadays [33].

These results provide new insight into the mass increase at relativistic velocities. An observer in the rest frame (S' frame) sees the mass of the electron (m) to increase at v with regard to the rest mass (m_0), which corresponds to the mass of the electron at $v =$ zero. This is imposed by the invariance of the energy in both frames mentioned above, in which both observers notice that the energy of the particle is the same, no matter the velocity they are travelling. This seems to be important to describing particle collisions at relativistic velocities, which should be frame independent.

Another insight even more interesting relates to the fact that some authors have considered the electron such as a massless particle [20]. In such a case, the energy of the electron at rest is only due to the wavelength related to the Schwinger electromagnetic wave [27]. Thus, the energy can only change if the electron is forced to move perpendicularly to the Compton circumference (see, for example, Figure 1 in [28]), since it is already at speed of light, leading to the helical motion. Based on that, hereafter the mass of the electron will be referred as the "*equivalent mass of the electron*", which is defined by

$$m^* \equiv \frac{h}{\mathrm{c}\lambda}, \tag{28}$$

and is related to the inertia to put the free electron at rest in motion along the direction perpendicular to the Compton circumference. At $v =$ zero, one can define the *equivalent rest mass of the electron* as $m_0^* \equiv h/c\lambda_C$.

The energy-moment relation given by Equation (15) can be rewritten by

$$E^2 = (p_\perp c)^2 + \left(p_\| c\right)^2 = (E_\perp)^2 + \left(E_\|\right)^2, \tag{29}$$

where $p_\perp$ and $p_\|$ are, respectively, the linear momentum transferred to the electron, which is related to the wavelength λ ($p_\perp = m^*v = hv/c\lambda,\ \mathrm{E}_\perp = p_\perp c = hv/\lambda$) and is perpendicular to the Compton circumference, and $p_\|$ which is related to the Compton wavelength λ_C ($p_\| = m_0^*c = h/\lambda_C, E_\| = p_\| c = hc/\lambda_C = m_0^*c^2$), due to the free electron at rest due travelling at the speed of light. Thus, the total energy is given by

$$E = \frac{hc}{\lambda} = \gamma\frac{hc}{\lambda_C} = \gamma m_0^* c^2 = m^* c^2, \tag{30}$$

which is the same as those given by Equations (16), (25), and (26) taking the m_0^* and m^* into account.

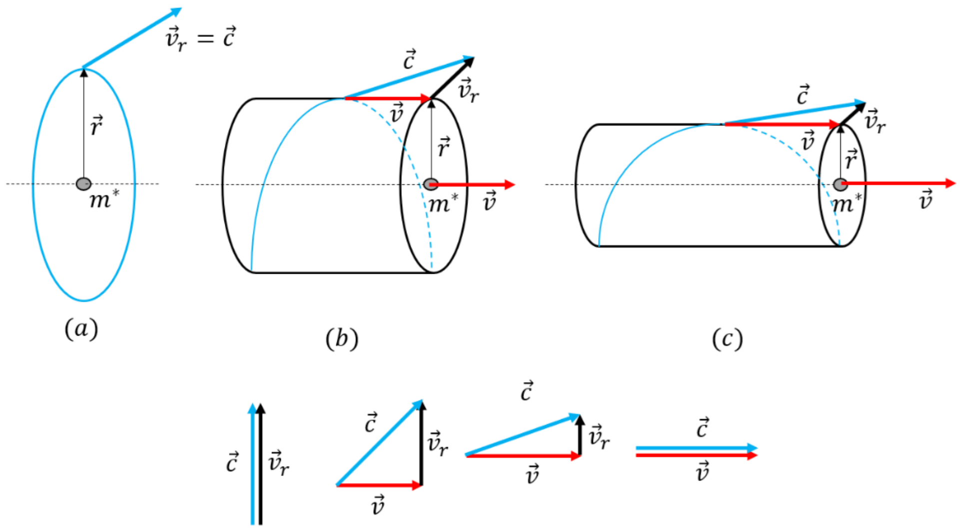

Figure 2. Effect of the relativistic velocity on the electron motion for the $r = r' = r_C\sqrt{1-(v/c)^2}$ case noticed by an observer in boosted frame (S). (a) Electron at rest in which its charge, due to Schwinger electromagnetic wave, is traveling at the speed of light with the equivalent center of mass at rest. In such a case $\vec{v}_r = \vec{c}$ and $r = r_C$. In (b) and (c) the equivalent center of mass of the electron is moving at two different velocities $\vec{v}$, which are perpendicular to the circular motion since it is already at the highest velocity possible, *i.e.*, $\vec{c}$, as shown in (a). P is the helical pitch and D is the distance traveled by the Schwinger wave, which is constant, as given by Equation (20). At $\vec{v} = \vec{c}$, $\vec{v}_r = 0$ and the helical motion vanishes, leading to a plane electromagnetic wave travelling at the speed of light along the longitudinal direction, *i.e.*, a photon (see arrows at right bottom).

Figure 2 displays the different situations saw by an observer in the boosted frame (S), in which helical electron motion is described for the case $r = r' = r_C\sqrt{1-(v/c)^2}$.

At $v =$ zero, the free electron is at rest and its energy is only related to the Schwinger electromagnetic wave ($E_0 = m_0^*c^2 = hc/\lambda_C$), since linear momentum of the equivalent mass is zero. As the velocity v increases, r decreases (r' in the rest frame decreases too) as shown in Figure 2 (b) and (c), but D remains constant no matter the v value. In such a case, the linear momentum is relevant and the equivalent mass of the electron plays a role in producing the helical motion. When the velocity becomes equal to the speed of light, the helical motion vanishes ($r = r' =$ zero) and D becomes equal to P, providing a plane electromagnetic wave with Compton wavelength, which has $f\lambda_C = c$ (see arrows at right bottom). For a better idea of the latter case, see the simulation available on the website provided in [42] or in Figure 1(a) reported by Natarajan [36], applying the limit $r \to$ zero.

Finally, in order to attend the expected spin and the *zitterbewegung* frequency values discussed by Williamson and van der Mark [13], one should define

$$r = r' = \frac{r_C}{2}\sqrt{1-(v/c)^2} = \frac{\lambda_C}{4\pi}\sqrt{1-(v/c)^2}, \tag{31}$$

which is a similar to the case described by the Equation (18), but taking into account the dependence with v.

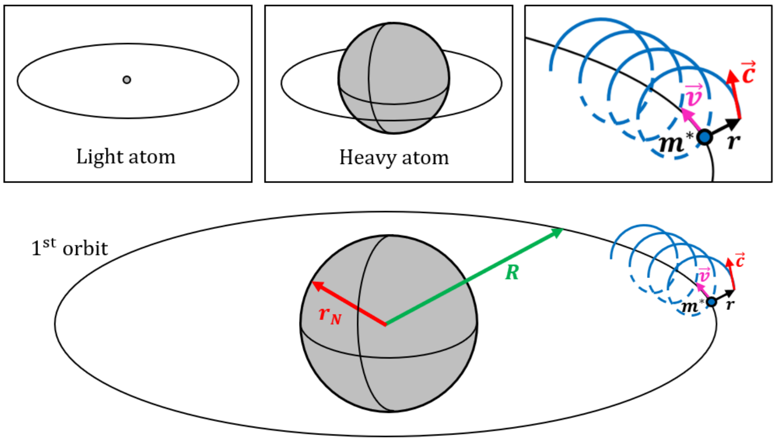

Figure 3. (Main panel) It is displayed the electron travelling in the first orbit with radius R in an atom with nucleus radius r_N. Upper left and middle insets compare light and heavy atoms. In the latter, the first orbit is much closer to the nucleus due to the number of protons (Z), which could cause instabilities in the atom due to the helical motion of the electron (see spiral blue lines at the right and in upper inset), due the relativistic effect on the Bohr and Compton radii. If $r \geq R - r_N$, electron charge will collide with the nucleus. The closer the first orbit is to the nucleus, the lower will be the atom stability.

2.2. Heavy Elements and Their Stabilities

Figure 3 displays the electron travelling in the first orbit with radius R around a nucleus of radius r_N with Z protons. As mentioned previously [19], [28], the electron charge moves in a helical way, while the electron center of mass follow the circular orbit.

In the hydrogen atom, R is the Bohr radius (r_B) and r is the Compton radius (r_C) or half of it [13], [30], $v \ll c$ ($v = v_B = \alpha c$), and the electron is far away from the nucleus ($r_N \ll r \ll R$), which is represented by the light atom in upper left part of the Figure 3.

In all cases, the electrical force acting on the equivalent mass of the electron, due to the Z protons in the nucleus, is given by

$$F = \frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{R^2} = \frac{m^* v^2}{R}, \tag{32}$$

which, after using the conservation of the angular momentum ($L = m^* vR = n\hbar$), provides the well-known radius and velocity for the first orbit [10], [37] as

$$v = \frac{Ze^2}{2\varepsilon_0 h} = \alpha c \mathrm{Z}, \tag{33}$$

and

$$R = \frac{\varepsilon_0 h^2}{\pi m^* Z e^2}, \tag{34}$$

respectively.

Table 1. It is displayed $R-r$ for the four cases described in this work. Third column defines the stability limit in which electron in the first orbit is about to collide with the nucleus. Z is the heaviest atom predicted from each case.

Case	$\boldsymbol{R-r}$	$\boldsymbol{R-r=r_N}$ **(fm)**	$\boldsymbol{Z}$
$r = r_C$	$r_C\left(\frac{1}{\alpha \mathrm{Z}}\sqrt{1-(\alpha \mathrm{Z})^2}-1\right)$	5.874	96.165
$r = \frac{r_C}{2}$	$\frac{r_C}{2}\left(\frac{2}{\alpha \mathrm{Z}}\sqrt{1-(\alpha \mathrm{Z})^2}-1\right)$	6.417	121.75
$r = r_C\sqrt{1-(v/c)^2}$	$r_C\left(\frac{1}{\alpha \mathrm{Z}}-1\right)\sqrt{1-(\alpha Z)^2}$	6.581	130.02
$r = \frac{r_C}{2}\sqrt{1-(v/c)^2}$	$\frac{r_C}{2}\left(\frac{2}{\alpha \mathrm{Z}}-1\right)\sqrt{1-(\alpha Z)^2}$	6.715	136.95

Taking the boosted frame (S) in the nucleus, the electron mass is travelling at the orbit velocity v. In such a case, the orbit radius can be rewritten taking into account the relativistic mass given by Equation (14), providing

$$R = \frac{r_B}{Z}\sqrt{1-(v/c)^2}, \tag{35}$$

where $r_B = \varepsilon_0 h^2/\pi m_0^* e^2$ and $r_C = \alpha r_B$ [19], [20], [28].

In order to know how far or close the electron in the first orbit is of the nucleus, one can use the following criteria to establish the stability of the atom

$$R - r \geq r_N, \tag{36}$$

where r_N is the radius of the atom with atomic number Z. If $R - r = r_N$, then the atom is unstable, since electron is about to collide with the nucleus. On the other hand, if $R - r > r_N$, electron does not collide with nucleus and the atom is stable.

As pointed out in the previous section, there are four models for r, which are summarized in Table 1.

The stability limit is determined by the dependencies of R, given in Equation (35), and r for each model described by Equations (17), (18), (19) and (31). Second column of the Table 1 displays how $R-r$ depends on r_C, α, and Z for the four cases.

In order to estimate which is the size of the nucleus that defines the above limit for each case, the data for r_N reported in [43] and [44] are used.

Figure 3(a) displays the behavior of Z as a function of the nucleus radius or the distance $R-r$ related to the cases discussed in this work. The experimental data [43], [44] are shown in Figure 3(b) along with the theoretical model by Smith *et al.* [45], [46] and the cases shown in table I, which allow one to determine the heaviest element for each model.

Black full line reported by Andrea [44] fits the collected experimental data by Angeli and Marinova [43], red dashed line is reported in Figure 8 of the reference by Smits *et al.* [45], [46], and other colored dashed lines are due $R-r$ discussed in this work. There is an additional model in which electron (mass + charge) is supposed to travel at the speed of light around the atom, providing $Z = 137 \approx 1/\alpha$ (Feynmanium) [41], [47], [48], but in such a model the size of the nucleus is not taken into account.

Considering the heaviest element known, recognized by IUPAC in 2016 as the Oganesson (Og), which has $Z = 118$ [49], the model for $r = r_C$ is unrealistic since it predicts the maximum $Z = 96$. On the other hand, comparing the other three models with the discovery of the element Oganesson recently [49], they look like more realistic.

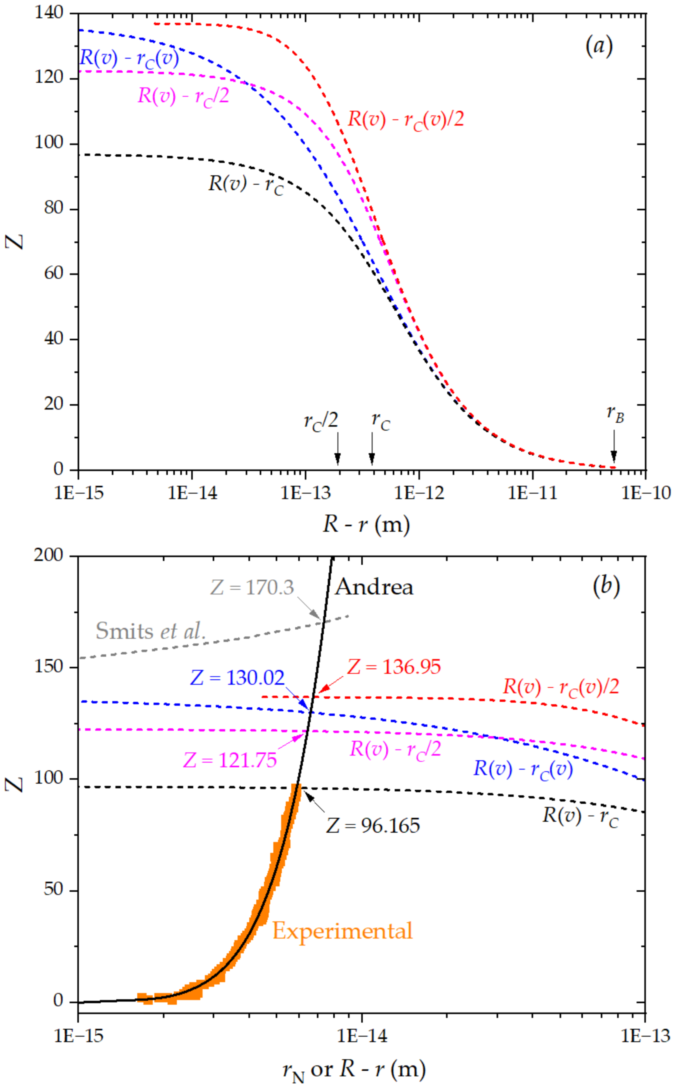

Figure 3. (*a*) Dependence of $R - r$ as a function of Z as predicted by Equations (17), (18), (19), and (31). (*b*) The models and experimental data are used to estimate the heaviest element. Orange square symbols are experimental data for r_N reported by Angeli and Marinova [43] and black full line is a fit reported by Andrea for these data [44]. Gray dashed line is reported by Smits *et al.* [45]. Other dashed lines are related to the four $R - r$ cases discussed in this work. The crossing points define the number of protons Z in which the heaviest element for each model is found. $r_B = 5.2918\, x\, 10^{-11}$ m, $r_C = 3.8616\, x\, 10^{-13}$ m, and $\alpha = 1/137.036$ are used to find $R - r$ behavior for all cases of this work.

3. Conclusions

This work provides a simple way to understand the relativistic effect on the wavelength and the energy-momentum relation for the electron. This arises from two different motions: the helical motion, considered to be related to the

Schwinger electromagnetic wave travelling at the speed of light, which is responsible for the electron charge, and the motion of the equivalent center of mass of the electron in the perpendicular direction to the helical motion. This occurs because the free electron at rest is already at the highest velocity possible, the speed of light. The unique way to increase its linear momentum is to change its velocity along the transverse direction of the Compton circumference.

Furthermore, the special relativity theory allows us to understand the electron structure in a simple way, providing support for the Bohr postulates demonstrated recently as a consequence of the helical motion of the electron.

The difference between Bohr and Compton radii compared to the nucleus radius of the atoms provides a new way to predict the heaviest element of the periodic table. The estimated values found to be between $Z = 122$ and 137, which are in agreement with the recent discovery of the Oganesson element ($Z = 118$).

References

[1] Dirac, P., *Principles of Quantum Mechanics*, Oxford University Press, 1930.

[2] Schrödinger, E., *Über die kraftefreie Bewegung in der relativistischen Quantenmechanik*. Sitzungsber. Preuss. Akad. Wiss. Berl. Phys.-math. Kl. (1930), *24*, 418–428.

[3] O'Connell, R. F., *Zitterbewegung* is not an Observable. Modern Physics Letters A. Vol. 26, No. 7 (2011) 469. https://doi.org/10.1142/S0217732311034967

[4] Kobakhidze A., Manning A., and Tureanu, A., Observable *Zitterbewegung* in curved spacetimes. Vol. 757, 2016, 84–91. https://doi.org/10.1016/j.physletb.2016.03.049

[5] Santos, I. U., The *Zitterbewegung* Electron Puzzle. Physics Essays, 2023, 36, 299–335. https://doi.org/10.4006/0836-1398-36.3.299

[6] A. Burinskii, The Dirac–Kerr–Newman Electron. Gravitation and Cosmology, 2008, Vol. 14, No. 2, pp. 109–122. https://doi.org/10.1134/S0202289308020011

[7] Burinskii, A., Gravitating Electron Based on Overrotating Kerr-Newman Solution. Universe 2022, 8, 553. https://doi.org/10.3390/universe8110553

[8] Sanchez, N. G., The classical-quantum duality of nature including gravity. International Journal of Modern Physics D. Vol. 28, No. 3 (2019) 1950055. https://doi.org/10.1142/S021827181950055X

[9] Oppenheim, J., A Postquantum Theory of Classical Gravity? Physical Review X 13, 041040 (2023). https://doi.org/10.1103/PhysRevX.13.041040

[10] Eisberg, R. Quantum Physics of Atoms, Molecules, Solids, Nuclei, and Particles, Second Edition. (John Wiley & Sons, 1985).

[11] Griffiths, D. J. Introduction to quantum mechanics I. (Prentice Hall, Inc., 1995).

[12] Barut, A. O. and Bracken, A. J., *Zitterbewegung* and the internal geometry of the electron, Phys. Rev. D 1981, *23*, 2454–2463. https://doi.org/10.1103/PhysRevD.23.2454

[13] Williamson, J. and van der Mark, M. "Is the electron a photon with toroidal topology?" Ann. Fond. Louis Broglie 1997, *22 (2)*, 133–158. https://fondationlouisdebroglie.org/AFLB-Web/en-annales-index.htm

[14] David Hestenes, Quantum mechanics from self-interaction, Found. Phys. 15(1) (1985) 63–87. https://doi.org/doi:10.1007/bf00738738

[15] Rivas, M. Kinematical Theory of Spinning Particles. December 2019. Available on: https://www.researchgate.net/publication/337744818

[16] Hu, Qiu-Hong. "The Nature of the Electron." Physics Essays 17, no. 4 (2004): 442–458. https://doi.org/10.4006/1.3025708.

[17] Hestenes, David. "*Zitterbewegung* in Quantum Mechanics." Foundations of Physics 40, no. 1 (2010): 1–54. https://doi.org/10.1007/s10701-009-9360-3

[18] Wilson, J. H., The Dirac electron nonpoint spatial structure generated over time by its rapidly oscillating charge shell. Phys. Essays 2015, 28 (1), 1–10. https://10.4006/0836-1398-28.1.1

[19] Rivas, M., The center of mass and center of charge of the electron. Journal of Physics: Conference Series (2015), 615, 012017. https://doi:10.1088/1742-6596/615/1/012017

[20] Celani, F., Di Tommaso, A. O., and Vassallo, G., The electron and Occam's razor, J. Condensed Matter Nucl. Sci. 25 (2017) 76–99. Available on: https://jcmns.scholasticahq.com/article/72460

[21] Consa, O., Helical Solenoid Model of the Electron, Prog. in Phys. 2018, 14, 80–89. https://www.researchgate.net/publication/326835988_Helical_Solenoid_Model_of_the_Electron.

[22] Di Tommaso, A. O. and Vassallo, G., Electron Structure, Ultra-dense Hydrogen and Low Energy Nuclear Reactions, J. Cond. Matt. Nucl. Sci. (2019), 29, 525–547. Available on: https://jcmns.scholasticahq.com/article/72529

[23] van Belle, J. L. The *zitterbewegung* interpretation of quantum mechanics, 2019. https://vixra.org/pdf/1901.0105vD.pdf

[24] Hestenes, D. *Zitterbewegung* structure in electrons and photons. 2020. https://doi.org/10.48550/arXiv.1910.11085

[25] Kovacs, A., Vassallo, G., O'Hara, P., Di Tommaso, A. O., Celani. F., Unified Field Theory and Occam's Razor: Simple Solutions to Deep Questions, World Scientific, 2022.

[26] Vassallo, G. and Kovacs, A., The Proton and Occam's Razor. Journal of Physics: Conference Series 2482 (2023) 012020. https://doi/10. 1088/1742-6596/2482/1/012020

[27] dos Santos, C. A. M. The Structure of the Electron Revealed by Schwinger Limits. June 2023 (under review). https://www.researchgate.net/publication/371909768

[28] dos Santos, C. A. M. and da Luz, M. S., Bohr Postulates Derived from the Toroidal Electron Model. Revista Mexicana de Física 70, 040201 (2024) 1–4. https://doi.org/10.31349/RevMexFis.70.040201

[29] dos Santos, C. A. M., da Luz, M. S, and Kovacs, A., Electron Mass-Charge Interaction in the Toroidal Electron Model. June 2024 (under review). https://www.researchgate.net/publication/381429630

[30] Rousselle, O., On the light-like and wave nature of elementary particles of matter, December 2023. https://www.researchgate.net/publication/376477256

[31] Kovacs, A., Zatelepin, V., Dmitry, B., Vassallo, G. and Sipilä, H., The proton's and neutron's internal structures: A simpler interpretation of modern nuclear measurements, 2024.

[32] Vassallo, G., Charge Clusters, Low Energy Nuclear Reactions and Electron Structure. April 2024. https://www.researchgate.net/publication/380122888

[33] Zitter Institute, https://www.zitter-institute.org/

[34] Ranzan, C., Physical Nature of Length Contraction (Part 2) How the Electron Elongates While Its Orbit Contracts. International Journal of High Energy Physics, 2019, 6(1), 1–12. https://doi/.org/10.11648/j.ijhep.20190601.11

[35] Ranzan, C., Mass-to-Energy Conversion, the Astrophysical Mechanism, Journal of High Energy Physics, Gravitation and Cosmology, 2019, 5, 520–551. https://doi/10.4236/jhepgc.2019.52030

[36] Natarajan, T. S. "Does the 'Complex' Wave Function in Quantum Mechanics Represent Anything 'Real' at All?" Quantum Speculations 5 (2023): 1–19. https://ijqf.org/wp-content/uploads/2023/06/QS2023v5n3p1.pdf

[37] Eisberg, Robert. Fundamentals of Modern Physics. 1st ed. New York: John Wiley & Sons, 1961.

[38] de Broglie, Louis-Victor. On the Theory of Quanta. 1924. Translated by A. F. Kracklauer. Paris: Annales de Physique. Available on: https://fondationlouisdebroglie.org/LDB-oeuvres/De_Broglie_Kracklauer.pdf

[39] Giorgio Vassalo, Charge Clusters, Low Energy Nuclear Reactions and Electron Structure. https://www.researchgate.net/publication/380122888

[40] During the course of writing this paper, we have noticed that this case was already considered by G. Vassalo and co-workers (see Equation (53) of the reference 40). Furthermore, simulations by these reserachers have also shown the decrease of r as a function of the velocity increasing, such as one can observe in Figure 1 and 2 of the references 20 and 40, respectively.

[41] dos Santos, C. A. M., da Luz, M. S., Oliveira, F. S., L. M. On the Fermi Gas, the Sommerfeld Fine Structure Constant, and the Electron-Electron Scattering in Conductors. Braz. J. Phys. 55, 2 (2025). https://doi.org/10.1007/s13538-024-01611-x

[42] https://en.wikipedia.org/wiki/Euler%27s_formula#/media/File:Rising_circular.gif

[43] Angeli, I. and Marinova, K. P., Table of experimental nuclear ground state charge radii: An update. Atomic Data and Nuclear Data Tables 99 (2013) 69–95. https://doi.org/10.1016/j.adt.2011.12.006

[44] Andrae, D., Finite nuclear charge density distributions in electronic structure calculations for atoms and molecules. Phys. Rep. 336 (6) (2000) 413–525. https://doi.org/10.1016/S0370-1573(00)00007-7

[45] Smits, O.R., Indelicato, P., Nazarewicz, W., Piibeleht, M., and Schwerdtfeger, P., Pushing the limits of the periodic table—A review on atomic relativistic electronic structure theory and calculations for the superheavy elements. Physics Reports 1035 (2023) 1–57. https://doi.org/10.1016/j.physrep.2023.09.004

[46] Pyykkö, P., A suggested periodic table up to $Z \leq 172$, based on Dirac–Fock calculations on atoms and ions. Phys. Chem. Chem. Phys., 2011, 13, 161–168.

[47] Chandrasekhar, S. On Stars, their evolution and their stability. Rev. Mod. Phys., 56, 2, Part I (1984).

[48] de Peralta, L. G., Poveda, L. A., and Poirier, B., Making relativistic quantum mechanics simple. Eur. J. Phys. 42 (2021) 055404.

[49] Öhrström, L. and Reedijk, J., Names and symbols of the elements with atomic numbers 113, 115, 117 and 118 (IUPAC Recommendations 2016). Pure Appl. Chem. 2016; 88(12): 1225–1229. http://dx.doi.org/10.1515/pac-2016-0501