J. Condensed Matter Nucl. Sci. 38 (2024) 338–356

Research Article

Thoughts on a Model of Vysotskii and Vysotskyy

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Abstract

Motivated by the recent proposal of Vysotstkii and Vysotskyy of the possibility of correlated coherent state evolution of an ion wave function in a low density magnetized plasma leading to low energy nuclear reactions, we have considered the construction of a dynamical squeezed state wave packet solution for a particle with a dynamic charge in a constant magnetic field. For a z-directed magnetic field, the dynamics simplify if the wave packet is constructed to have the same dynamics in x and y. From the solution developed, evolution equations are obtained for the first and second moments. In the case of the first moments, the dynamics that result are Newton's laws for a particle in a magnetic field. The second moments evolve according to what would be expected for a squeezed state solution, except that the breathing frequency is reduced by a factor of two. The uncertainty product for the squeezed state is a Schrodinger-Robertson relation. The amount of squeezing required for the fluctuation energy to be sufficient to result in nuclear reactions is extremely large, and such a wave packet would be quite fragile in the sense that collisions with other particles would prevent organized "breathing" mode evolution except at very low plasma density. Following the proposal of Vysotskii and Vysotskyy, we constructed generalization of the squeezed state wave packet in which an excited state SHO eigenfunction is squeezed, and find that the evolution of the squeezing parameters is similar as for a squeezed state solution. The uncertainty principle is of the form of the Schrodinger-Robertson relation, but with a prefactor that substantially increases the uncertainty. An issue that was identified in the proposed scheme is localization associated with electron ionization and recombination that has the potential to spoil the delocalization and phase of the wave packet. It is noted that in a minor modification of the scheme non-classical energy exchange may be possible for charged particles traveling through a dynamic and spatially inhomogeneous magnetic field, as long as the changes in the field as seen by the particle are non-adiabatic relative to the squeezing dynamics.

Keywords: Squeezed states, correlated coherent states, non-classical heating, low-density magnetized plasma

1. Introduction

Vysotskii and Vysotskyy have proposed that sequential ionization and recombination of hydrogen isotopes in a magnetized plasma can produce correlated coherent states at a temperature (10-20 eV) that can result in nuclear reactions requiring high energy (10 keV) [1], [2] (see also [3]–[5]). Such a result is unexpected, and worth some thought. This provides motivation for the construction of a squeezed state wave function that is relevant to the problem, as well as the construction of a generalization based on the squeezing of excited eigenfunctions of the simple harmonic oscillator.

In essence, we were interested in better understanding the model. In a quantum class taught at MIT, we make use of a configuration space representation of squeezed state wave packets, as they are easy to work with and don't require operator algebra. We were interested in developing explicit wave packet solutions, in part to develop an example that might at some point be used in class, but which in the process would hopefully provide some clarity as to the physics of interest in the model.

The model that results provides for a "clean" and explicit solution. The amount of squeezing required for a proton to gain 10 keV of fluctuation energy is extreme (as noted by Vysotskii and Vysotskyy [2]), and the associated wave packet becomes very delocalized during the breathing, which makes it fragile in the sense of being quite sensitive to collisions with neighboring particles.

Following on the suggestion of Vysotskii and Vysotskyy that the squeezing of a highly excited simple harmonic oscillator (SHO) eigenstate would work better [2], in the Appendix we consider the construction of a wave packet made from the squeezing of a highly excited eigenfunction.

2. Schrodinger Equation

Vysotskii and Vysotskyy have proposed modeling ionization and recombination of a hydrogen atom through the use of a time-dependent charge [1], [2]. While it would seem better to model the system using a many-particle wave function to describe the proton and electrons involved, our focus here is on the squeezed state part of the problem. So we will follow Vysotskii and Vysotskyy and work with a time-dependent charge.

The Schrodinger equation in the nonrelativistic approximation (and ignoring spin effects) is

$$i\hbar\frac{\partial}{\partial t}\Psi(\mathbf{r},t) \;=\; \frac{|\mathbf{p}-q(t)\mathbf{A}|^2}{2m}\Psi(\mathbf{r},t) \tag{1}$$

The magnetic field is static, and is described by the vector potential

$$\mathbf{A} \;=\; \frac{1}{2}\hat{\mathbf{i}}_z B_0\times\mathbf{r} \;=\; \frac{1}{2}\begin{vmatrix}\hat{\mathbf{i}}_x & \hat{\mathbf{i}}_y & \hat{\mathbf{i}}_z\\ 0 & 0 & B_0\\ x & y & z\end{vmatrix} \;=\; \frac{B_0}{2}\left(\hat{\mathbf{i}}_y x-\hat{\mathbf{i}}_x y\right) \tag{2}$$

From this vector potential we compute the magnetic field to be

$$\mathbf{B} \;=\; \nabla\times\mathbf{A} \;=\; \begin{vmatrix}\hat{\mathbf{i}}_x & \hat{\mathbf{i}}_y & \hat{\mathbf{i}}_z\\ \partial_x & \partial_y & \partial_z\\ -B_0 y/2 & B_0 x/2 & 0\end{vmatrix} \;=\; \hat{\mathbf{i}}_z B_0 \tag{3}$$

We can use this to expand out the Schrodinger equation according to

$$\begin{aligned} i\hbar\frac{\partial}{\partial t}\Psi(\mathbf{r},t) &= \left(\frac{|\mathbf{p}|^2}{2m}-q(t)\frac{\mathbf{A}\cdot\mathbf{p}}{m}+q^2(t)\frac{|\mathbf{A}|^2}{2m}\right)\Psi(\mathbf{r},t)\\ &= \left(\frac{|\mathbf{p}|^2}{2m}-q(t)\frac{A_x p_x+A_y p_y}{m}+q^2(t)\frac{A_x^2+A_y^2}{2m}\right)\Psi(\mathbf{r},t)\end{aligned}$$

$$= \left(\frac{|\mathbf{p}|^2}{2m} - B_0 q(t) \frac{-yp_x + xp_y}{2m} + B_0^2 q^2(t) \frac{x^2+y^2}{8m} \right) \Psi(\mathbf{r}, t)$$

$$= \left(- \frac{\hbar^2}{2m} (\partial_x^2 + \partial_y^2 + \partial_z^2) - i \frac{\hbar B_0 q(t)}{2m} (y \partial_x - x \partial_y) + \frac{B_0^2 q^2(t)}{8m} (x^2 + y^2) \right) \Psi(\mathbf{r}, t) \tag{4}$$

The goal is to study a time-dependent wave function solutions for this model.

3. Correlated Coherent States

Correlated coherent states were proposed by Dodonov and coworkers as a generalization of squeezed states [6]. Such states satisfy the Schrodinger-Robertson uncertainty relation

$$\sigma_A \sigma_B - \sigma_{AB}^2 \ = \ \sigma_A \sigma_B (1 - r^2) \ \geq \ \frac{1}{4} \left| \left\langle \left[\hat{A}, \hat{B} \right] \right\rangle \right|^2 \tag{5}$$

with minimum uncertainty, where $\hat{A}$ and $\hat{B}$ are operators of interest for a particular problem. In the event that these operators are taken to be position and momentum operators, then this reduces to

$$(\Delta x)^2 (\Delta p)^2 (1 - r^2) \ \geq \ \frac{\hbar^2}{4} \tag{6}$$

with r the correlation coefficient according to

$$r \ = \ \frac{\sigma_{AB}}{\sqrt{\sigma_A \sigma_B}} \tag{7}$$

In their papers, Dodonov and coworkers focus on a particular configuration space construction relevant to position and momentum operators, such as [6]–[9]

$$\Psi_\beta(x|r, \eta) \ = \ N_\beta \exp \left\{ - \frac{x^2}{4\eta} \left(1 - i \frac{r}{\sqrt{1 - r^2}} \right) + \frac{\beta x}{\eta} \right\} \tag{8}$$

From our perspective, such a state could be considered to be a snapshot of a squeezed wave packet solution at a particular time, which suggests that it would be possible to develop a squeezed state wave packet solution based on

$$\Psi(x, t) \ = \ N_{\beta(t)} \exp \left\{ - \frac{x^2}{4\eta(t)} \left(1 - i \frac{r(t)}{\sqrt{1 - r^2(t)}} \right) + \frac{\beta(t) x}{\eta(t)} \right\} \tag{9}$$

A big advantage of a configuration space description of a squeezed state wave packet is that it is very convenient for calculations, where the equivalent calculations based on operator algebra would be much harder.

The solution that we consider in the following sections could be considered to be a three-dimensional generalization according to

$$\Psi(x,y,z,t) \ = \ N_{\beta_x(t)} \exp\left\{ - \frac{x^2}{4\eta_x(t)} \left(1 - i \frac{r_x(t)}{\sqrt{1-r_x^2(t)}} \right) + \frac{\beta_x(t) x}{\eta_x(t)} \right\}$$

$$N_{\beta_y(t)} \exp\left\{ - \frac{y^2}{4\eta_y(t)} \left(1 - i \frac{r_y(t)}{\sqrt{1-r_y^2(t)}} \right) + \frac{\beta_y(t) x}{\eta_y(t)} \right\}$$

$$N_{\beta_z(t)} \exp\left\{ - \frac{z^2}{4\eta_z(t)} \left(1 - i \frac{r_z(t)}{\sqrt{1-r_z^2(t)}} \right) + \frac{\beta_z(t) x}{\eta_z(t)} \right\} \tag{10}$$

There would be no difficulty in principle using this representation as a starting place for solving the time-dependent Schrodinger equation for a nonrelativistic particle in a magnetic field. However, based on our previous experience with this kind of problem, calculations based on this kind of wave function written in a different form leads to dynamical equations for the parameters which are more intuitive.

4. General Squeezed State Wave Packet

We start with a general squeezed state wave packet given by

$$\Psi(\mathbf{r},t) \ = \ C(t) e^{-i\Theta(t)} e^{iP_x(t)(x-X(t))/\hbar} e^{iP_y(t)(y-Y(t))/\hbar} e^{iP_z(t)(z-Z(t))/\hbar}$$

$$e^{-\alpha_{xx}(t)(x-X(t))^2/2} e^{-\alpha_{xy}(t)(x-X(t))(y-Y(t))} e^{-\alpha_{yy}(t)(y-Y(t))^2/2} e^{-\alpha_{zz}(t)(z-Z(t))^2/2} \tag{11}$$

The center of the wave packet is located at

$$\langle \mathbf{r} \rangle \ = \ \hat{\mathbf{i}}_x X(t) + \hat{\mathbf{i}}_y Y(t) + \hat{\mathbf{i}}_z Z(t) \tag{12}$$

and the mean momentum is

$$\langle \hat{\mathbf{p}} \rangle \ = \ \hat{\mathbf{i}}_x P_x(t) + \hat{\mathbf{i}}_y P_y(t) + \hat{\mathbf{i}}_z P_z(t) \tag{13}$$

Distortion of the wave packet (or fluctuations) is modeled using the α_{xx}, α_{yy}, α_{xy} and α_{zz} parameters. For example, if the real parts of these parameters are big, then the wave packet is well localized.

To maintain normalization, we require that

$$\int \Psi^*(\mathbf{r},t)\Psi(\mathbf{r},t) d^3\mathbf{r} \ = \ 1 \tag{14}$$

The integrals can be done analytically, and we end up with

$$C(t) \ = \ \frac{1}{\pi^{3/4}} \left(\mathrm{Re}\{\alpha_{xx}(t)\}\mathrm{Re}\{\alpha_{yy}(t)\} - \mathrm{Re}\{\alpha_{xy}(t)\}^2 \right)^{1/4} \left(\mathrm{Re}\{\alpha_{zz}(t)\} \right)^{1/4} \tag{15}$$

5. Independence

The x, y degrees of freedom and the z degrees of freedom are independent in this model. We can write the Schrodinger equation as

$$i\hbar\partial_t \Psi(x,y,z,t) \ = \ \left(H_{xy} + H_z \right) \Psi(x,y,z,t) \tag{16}$$

and the wave function as

$$\Psi(x,y,z,t) \ = \ \psi_{xy}(x,y,t)\psi_z(z,t) \tag{17}$$

Then separation can lead to

$$i\hbar\partial_t \psi_{xy}(x,y,t) \ = \ H_{xy}\psi_{xy}(x,y,t) \tag{18}$$

$$i\hbar\partial_t \psi_z(z,t) \ = \ H_z\psi_z(z,t) \tag{19}$$

These can be written out as

$$i\hbar\frac{\partial}{\partial t}\psi_{xy}(x,y,t) = \left\{ -\frac{\hbar^2}{2m}(\partial_x^2 + \partial_y^2) - i\frac{\hbar B_0 q(t)}{2m}(y\partial_x - x\partial_y) + \frac{B_0^2 q^2(t)}{8m}(x^2+y^2) \right\} \psi_{xy}(x,y,t) \tag{20}$$

$$i\hbar\frac{\partial}{\partial t}\psi_z(z,t) = -\frac{\hbar^2}{2m}\partial_z^2\psi_z(z,t) \tag{21}$$

6. Evolution Equations for the z-Degrees of Freedom

Since the problem in z is much simpler, we focus on it first. Using Mathematica we plug in

$$\psi_z(z,t) \ = \ \left(\frac{\mathrm{Re}\{\alpha_{zz}\}}{\pi} \right)^{1/4} e^{-i\Theta_z(t)} e^{iP_z(t)(z-Z(t))/\hbar} e^{-\alpha_{zz}(t)(z-Z(t))^2/2} \tag{22}$$

Mathematica gives

$$\begin{aligned}
&-\frac{1}{2}i\hbar z^2\alpha'_{zz}(t) + i\hbar z Z(t)\alpha'_{zz}(t) - \frac{1}{2}i\hbar Z(t)^2\alpha'_{zz}(t) + \frac{i\hbar\alpha'_{zz}(t)}{4\alpha_{zz}(t)} + i\hbar z\alpha_{zz}(t)Z'(t) \\
&- i\hbar\alpha_{zz}(t)Z(t)Z'(t) + \hbar\Theta'(t) - zP'_z(t) + Z(t)P'_z(t) + P_z(t)Z'(t) \\
&= -\frac{\hbar^2 z^2\alpha_{zz}(t)^2}{2m} + \frac{\hbar^2 z\alpha_{zz}(t)^2 Z(t)}{m} - \frac{\hbar^2\alpha_{zz}(t)^2 Z(t)^2}{2m} + \frac{\hbar^2\alpha_{zz}(t)}{2m} \\
&+ \frac{i\hbar z\alpha_{zz}(t)P_z(t)}{m} - \frac{i\hbar\alpha_{zz}(t)P_z(t)Z(t)}{m} + \frac{P_z(t)^2}{2m}
\end{aligned} \tag{23}$$

We can match terms proportional to z^0, z^1 and z^2 individually to get constraints.

Two of the constraints correspond to Newton's laws for propagation in free space:

$$\frac{d}{dt}Z(t) = \frac{P_z(t)}{m} \tag{24}$$

$$\frac{d}{dt}P_z(t) = 0 \tag{25}$$

We know that a wave packet spreads in free space. This is due to the uncertainty principle, where localization in space leads to a spread of momentum, which means that the wave packet spreads (there are no forces in free space to keep it localized). Within the model this is accounted for by

$$i\hbar\frac{d}{dt}\alpha_{zz}(t) \;=\; \frac{\hbar^2}{m}\alpha_{zz}^2(t) \tag{26}$$

An exact solution is possible. However, the focus of our arguments is concerned with the x, y degrees of freedom, so we will not pursue this here.

The phase Θ_z is required to satisfy the Schrodinger equation, and satisfies

$$\hbar\frac{d}{dt}\Theta_z(t) \;=\; -\frac{P_z^2(t)}{2m} + \frac{\hbar^2\alpha_{zz}(t)}{4m} \tag{27}$$

This can be solved exactly with no difficulty. However, the phase factor is not important in connection with the issues of interest.

The expectation value of the energy is

$$\langle H_z\rangle \;=\; \frac{P_z^2(t)}{2m} + \frac{\hbar^2}{4m}\frac{|\alpha_{zz}(t)|^2}{\mathrm{Re}\{\alpha_{zz}(t)\}} \tag{28}$$

This is both interesting and instructive. The expectation value of the energy is split into two pieces: one is the classical kinetic energy, and the other is the (non-classical) kinetic energy associated with the localization of the wave packet. Note also that the expectation value of the energy associated with the z-degree of freedom is constant in time, since

$$\begin{aligned}\frac{d}{dt}\frac{|\alpha_{zz}(t)|^2}{\mathrm{Re}\{\alpha_{zz}(t)\}} &= \frac{2}{(\alpha_{zz}(t)+\alpha_{zz}^*(t))^2}\left((\alpha_{zz}^*(t))^2\frac{d}{dt}\alpha_{zz}(t) + \alpha_{zz}^2(t)\frac{d}{dt}\alpha_{zz}^*(t)\right)\\ &= \frac{2}{(\alpha_{zz}(t)+\alpha_{zz}^*(t))^2}\left((\alpha_{zz}^*(t))^2\frac{\hbar^2}{im}\alpha_{zz}^2(t) - \alpha_{zz}^2(t)\frac{\hbar^2}{im}(\alpha_{zz}^*(t))^2\right)\\ &= 0\end{aligned} \tag{29}$$

7. Evolution Equations for the x and y Degrees of Freedom

For the next part of the problem we start with

$$\begin{aligned}\psi_{xy}(x,y,t) \;=\; &\frac{1}{\pi^{1/2}}\left(\mathrm{Re}\{\alpha_{xx}\}\mathrm{Re}\{\alpha_{yy}\} - \mathrm{Re}\{\alpha_{xy}\}^2\right)^{1/4} e^{-i\Theta_{xy}(t)}e^{iP_x(t)(x-X(t))/\hbar}e^{iP_y(t)(y-Y(t))/\hbar}\\ &e^{-\alpha_{xx}(t)(x-X(t))^2/2}e^{-\alpha_{yy}(t)(y-Y(t))^2/2}e^{-\alpha_{xy}(t)(x-X(t))(y-Y(t))}\end{aligned} \tag{30}$$

This is plugged into the Schrodinger equation

$$i\hbar\frac{\partial}{\partial t}\psi_{xy}(x,y,t) \;=\; \left(-\frac{\hbar^2}{2m}(\partial_x^2+\partial_y^2) - i\frac{\hbar B_0 q(t)}{2m}(y\partial_x - x\partial_y) + \frac{B_0^2 q^2(t)}{8m}(x^2+y^2)\right)\psi_{xy}(x,y,t) \tag{31}$$

As might be expected, quite a few terms are generated. It is possible to match terms to develop a set of constraints.

As expected, the first-order constraints are Newton's law equations according to

$$\frac{d}{dt}X(t) = \frac{P_x(t)}{m} + \frac{B_0 q(t)}{2m}Y(t) \tag{32}$$

$$\frac{d}{dt}P_x(t) = -\frac{B_0^2 q^2(t)}{4m}X(t) + \frac{B_0 q(t)}{2m}P_y(t) \tag{33}$$

$$\frac{d}{dt}Y(t) = \frac{P_y(t)}{m} - \frac{B_0 q(t)}{2m}X(t) \tag{34}$$

$$\frac{d}{dt}P_y(t) = -\frac{B_0^2 q^2(t)}{4m}Y(t) - \frac{B_0 q(t)}{2m}P_x(t) \tag{35}$$

It is possible to make use of the kinetic momenta variables

$$\begin{aligned}\Pi_x(t) \;&=\; P_x(t) - q(t)A_x(t) \;=\; P_x(t) + \frac{B_0 q(t)}{2}Y(t)\\ \Pi_y(t) \;&=\; P_y(t) - q(t)A_y(t) \;=\; P_y(t) - \frac{B_0 q(t)}{2}X(t)\end{aligned} \tag{36}$$

to write these equations in terms of Newton's equations with the Lorentz force law

$$\begin{aligned}\frac{d}{dt}\mathbf{R}_\perp &= \frac{\mathbf{\Pi}_\perp}{m}\\ \frac{d}{dt}\mathbf{\Pi}_\perp &= q\frac{\mathbf{\Pi}_\perp}{m}\times\mathbf{B}\end{aligned} \tag{37}$$

as expected.

Also from matching terms we can develop evolution equations for the squeezing parameters

$$i\hbar\frac{d}{dt}\alpha_{xx}(t) = \frac{\hbar^2}{m}\alpha_{xx}^2(t) + \frac{\hbar^2}{m}\alpha_{xy}^2(t) + i\frac{\hbar B_0 q(t)}{m}\alpha_{xy}(t) - \frac{B_0^2 q^2(t)}{4m} \tag{38}$$

$$i\hbar\frac{d}{dt}\alpha_{yy}(t) = \frac{\hbar^2}{m}\alpha_{xy}^2(t) + \frac{\hbar^2}{m}\alpha_{yy}^2(t) - i\frac{\hbar B_0 q(t)}{m}\alpha_{xy}(t) - \frac{B_0^2 q^2(t)}{4m} \tag{39}$$

$$i\hbar\frac{d}{dt}\alpha_{xy}(t) = \frac{\hbar^2}{m}\alpha_{xy}(t)\alpha_{yy}(t) + \frac{\hbar^2}{m}\alpha_{xx}(t)\alpha_{xy}(t) - i\frac{\hbar B_0 q(t)}{2m}\alpha_{xx}(t) + i\frac{\hbar B_0 q(t)}{2m}\alpha_{yy}(t) \tag{40}$$

Unfortunately, these coupled equations are sufficiently complicated in general that we are motivated to look for a simplification.

Mathematica gives a complicated evolution equation for $\Theta_{xy}(t)$ that we have not developed a simplification for, so we will not pursue the issue here.

8. Simplification

It is possible to simplify the model by focusing on a specific case. If we look at the evolution equations for the squeezing parameters, we see that if assume that

$$\begin{aligned} \alpha_{xx}(t) &= \alpha_{yy}(t) \\ \alpha_{xy}(t) &= 0 \end{aligned} \tag{41}$$

then the evolution equations for the squeezing parameters reduce to

$$i\hbar \frac{d}{dt}\alpha_{xx}(t) = \frac{\hbar^2}{m}\alpha_{xx}^2(t) - \frac{B_0^2 q^2(t)}{4m} \tag{42}$$

$$i\hbar \frac{d}{dt}\alpha_{yy}(t) = \frac{\hbar^2}{m}\alpha_{yy}^2(t) - \frac{B_0^2 q^2(t)}{4m} \tag{43}$$

$$i\hbar \frac{d}{dt}\alpha_{xy}(t) = 0 \tag{44}$$

In this limit the problem simplifies and separates. For the solution we can write

$$\begin{aligned} \psi_{xy}(x,y,t) \ \to \ & \frac{1}{\pi^{1/2}} \left(\mathrm{Re}\{\alpha_{xx}\}\mathrm{Re}\{\alpha_{yy}\}\right)^{1/4} e^{-i\Theta_{xy}(t)} e^{iP_x(t)(x-X(t))/\hbar} e^{iP_y(t)(y-Y(t))/\hbar} \\ & e^{-\alpha_{xx}(t)(x-X(t))^2/2} e^{-\alpha_{yy}(t)(y-Y(t))^2/2} \end{aligned} \tag{45}$$

The expectation value of the Hamiltonian is

$$\begin{aligned} \langle H_{xy} \rangle &= \left\langle -\frac{\hbar^2}{2m}(\partial_x^2 + \partial_y^2) - i\frac{\hbar B_0 q(t)}{2m}(y\partial_x - x\partial_y) + \frac{B_0^2 q^2(t)}{8m}(x^2+y^2) \right\rangle \\ &= \frac{P_x^2(t)}{2m} + \frac{P_y^2(t)}{2m} + \frac{B_0 q(t)}{2m}\bigg(-X(t)P_y(t) + Y(t)P_x(t) \bigg) + \frac{B_0^2 q^2(t)}{8m}\bigg(X^2(t) + Y^2(t) \bigg) \\ &\quad + \frac{\hbar^2 |\alpha_{xx}(t)|^2}{4m\mathrm{Re}\{\alpha_{xx}(t)\}} + \frac{\hbar^2 |\alpha_{yy}(t)|^2}{4m\mathrm{Re}\{\alpha_{yy}(t)\}} + \frac{B_0^2 q^2(t)}{16m\mathrm{Re}\{\alpha_{xx}(t)\}} + \frac{B_0^2 q^2(t)}{16m\mathrm{Re}\{\alpha_{yy}(t)\}} \end{aligned} \tag{46}$$

The classical contribution to the expectation value of the Hamiltonian is precisely what we would expect. The last line is the fluctuation energy, which is made up of the kinetic energy fluctuations and the potential energy fluctuations. In the event that the time dependent charge is either $q(t) = 0$ or $q(t) = |e|$ the total fluctuation energy is constant.

The fluctuation energy changes when the charge changes in this model due to the potential energy contribution to the fluctuation energy.

The classical energy (the first line on the RHS of equation (46)) is constant assuming that the magnetic field and charge are constant. Also, the fluctuation energy (the second line on the RHS of equation (46)) is also constant. This can be verified from a direct calculation of the x contribution

$$\frac{d}{dt}\left\{ \frac{\hbar^2|\alpha_{xx}(t)|^2}{4m\mathrm{Re}\{\alpha_{xx}(t)\}} + \frac{B_0^2 q^2}{16m\mathrm{Re}\{\alpha_{xx}(t)\}} \right\} \ = \ 0 \tag{47}$$

which involves a fair amount of algebra, that we have verified works within this model.

9. Solution for the Squeezing Parameter

It is possible to develop solutions for the evolution equations for the squeezing parameters. For example, suppose that we start with

$$i\hbar\frac{d}{dt}\alpha_{xx}(t) \ = \ \frac{\hbar^2}{m}\alpha_{xx}^2(t) - \frac{B_0^2 q^2}{4m} \tag{48}$$

with $\alpha_{xx}(0)$ given for the special case when $q(t) = q = |e|$. We can solve and write

$$\frac{\hbar\alpha_{xx}(t)}{m\omega_a} \ = \ \frac{\frac{\hbar\alpha_{xx}(0)}{m\omega_a}\cos(\omega_a t) + i\sin(\omega_a t)}{\cos(\omega_a t) + i\frac{\hbar\alpha_{xx}(0)}{m\omega_a}\sin(\omega_a t)} \tag{49}$$

where ω_a is

$$\omega_a \ = \ \frac{B_0 q}{2m} \tag{50}$$

Note that squeezing in this system does not work like in a normal one-dimensional SHO, where the relevant frequency for squeezing is the same as the oscillator frequency. In this system the oscillator frequency ω_0 is the cyclotron frequency

$$\omega_0 \ = \ \frac{B_0 q}{m} \tag{51}$$

and the frequency relevant to squeezing ω_a is half of this

$$\omega_a \ = \ \frac{1}{2}\omega_0 \tag{52}$$

This means that if a wave packet is localized at one point in the cycle in the $x-y$ plane, that it reforms at the same point on the next cycle (schematic in Figure 1).

The cyclotron frequencies of interest in the model of Vysotskii and Vysotskyy are

$$\omega_0 \ = \ 2\pi \ \times \ 1.54\,\mathrm{MHz} \quad (1000\ \mathrm{Oe}) \qquad \omega_0 \ = \ 2\pi \ \times \ 4.61\,\mathrm{MHz} \quad (3000\ \mathrm{Oe}) \tag{53}$$

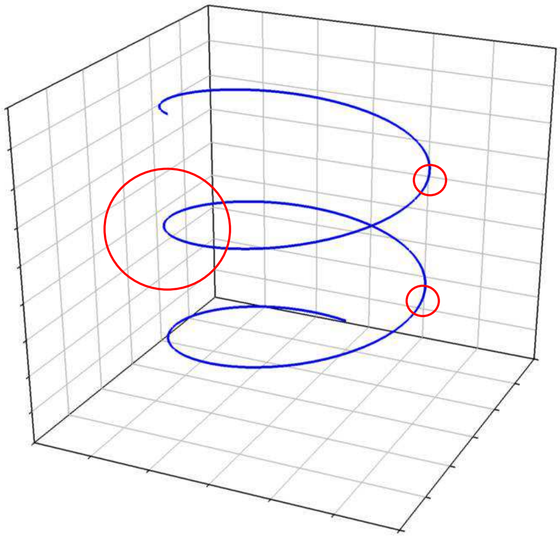

Figure 1. Schematic of squeezing in connection to particle motion in a magnetic field.

10. Wave Packet Energy

It is of interest to examine the energy of the wave packet, which is conserved when the magnetic field and charge are constant. For simplicity, we consider the situation when

$$q(t) \;\rightarrow\; q \;=\; |e| \tag{54}$$

The expectation value of the energy can be divided into classical and non-classical terms according to

$$\langle H\rangle \;=\; \langle H\rangle_c + \langle T\rangle_{nc} + \langle V\rangle_{nc} \tag{55}$$

where based on the calculation above we can write for the classical energy

$$\langle H\rangle_c \;=\; \frac{P_x^2(t)}{2m} + \frac{P_y^2(t)}{2m} + \frac{B_0 q}{2m}\Big(- X(t)P_y(t) + Y(t)P_x(t)\Big) + \frac{B_0^2 q^2}{8m}\Big(X^2(t) + Y^2(t)\Big) \tag{56}$$

The non-classical kinetic energy is

$$\langle T\rangle_{nc} \;=\; +\frac{\hbar^2|\alpha_{xx}(t)|^2}{4m\mathrm{Re}\{\alpha_{xx}(t)\}} + \frac{\hbar^2|\alpha_{yy}(t)|^2}{4m\mathrm{Re}\{\alpha_{yy}(t)\}} \tag{57}$$

and the non-classical potential energy is

$$\langle V\rangle_{nc} \;=\; \frac{B_0^2 q^2}{16m\mathrm{Re}\{\alpha_{xx}(t)\}} + \frac{B_0^2 q^2}{16m\mathrm{Re}\{\alpha_{yy}(t)\}} \tag{58}$$

Since the classical and non-classical degrees of freedom are independent, we can find solutions for $X(t)$ and $Y(t)$ that satisfy

$$\langle H \rangle_c \sim 20 \text{ eV} \tag{59}$$

while at the same time developing solutions for $\alpha_{xx}(t)$ and $\alpha_{yy}(t)$ that satisfy

$$\langle T \rangle_{nc} + \langle V \rangle_{nc} \sim 10 \text{ keV} \tag{60}$$

11. Localization

The fluctuation energy oscillates between kinetic energy and potential energy, as can be understood from the time-dependence of the solution for $\alpha_{xx}(t)$ given above. The kinetic energy is highest when the wave function is most localized. For the kinetic energy to reach 10 keV, the localization can be estimated starting from

$$\frac{(\Delta p_x)^2}{2m} = 5 \text{ keV} \tag{61}$$

since there are 2 degrees of freedom. Since the maximally squeezed wave packet is Gaussian in this model we have

$$\Delta x \Delta p = \frac{\hbar}{2} \tag{62}$$

The localization in x satisfies

$$\frac{\hbar^2}{8m(\Delta x)^2} = 5 \text{ keV} \tag{63}$$

This is evaluated to give

$$\Delta x \sim 3.2 \times 10^{-4} \text{ Angstrom} \tag{64}$$

12. Delocalization

After the squeezed wave packet is localized, it spreads out. Of interest is to estimate how big the spread is. To proceed we recall that

$$\frac{\hbar \alpha_{xx}(t)}{m\omega_a} = \frac{\frac{\hbar \alpha_{xx}(0)}{m\omega_a} \cos(\omega_a t) + i \sin(\omega_a t)}{\cos(\omega_a t) + i \frac{\hbar \alpha_{xx}(0)}{m\omega_a} \sin(\omega_a t)} \tag{65}$$

So, the maximum and minimum values are

$$\left(\frac{\hbar \alpha_{xx}(t)}{m\omega_a} \right)_{max} = \frac{\hbar \alpha_{xx}(0)}{m\omega_a} \qquad \left(\frac{\hbar \alpha_{xx}(t)}{m\omega_a} \right)_{min} = \frac{m\omega_a}{\hbar \alpha_{xx}(0)} \tag{66}$$

or

$$\left(\alpha_{xx}(t)\right)_{min} = \frac{m^2\omega_a^2}{\hbar^2\alpha_{xx}(0)} \tag{67}$$

It follows that

$$(\Delta x)^2_{min} = \frac{1}{2(\alpha_{xx})_{max}} = \frac{1}{2\alpha_{xx}(0)} \tag{68}$$

$$(\Delta x)^2_{max} = \frac{1}{2(\alpha_{xx})_{min}} = \frac{1}{2}\frac{\hbar^2}{m^2\omega_a^2}\alpha_{xx}(0) \tag{69}$$

We can use this to write

$$\frac{(\Delta x)_{max}}{(\Delta x)_{min}} = \frac{\hbar}{m\omega_a}\alpha_{xx}(0) = \frac{\hbar}{2m\omega_a(\Delta x)^2_{min}} \tag{70}$$

We recall that

$$\frac{\hbar^2}{8m(\Delta x)^2_{min}} = 5 \text{ keV} \tag{71}$$

So we have

$$\frac{(\Delta x)_{max}}{(\Delta x)_{min}} = \frac{20 \text{ keV}}{\hbar\omega_a} = \frac{40 \text{ keV}}{\hbar\omega_0} \tag{72}$$

We can evaluate this for the two magnetic field strengths to obtain

$$\frac{(\Delta x)_{max}}{(\Delta x)_{min}} = 6.26 \times 10^{12} \quad (1000 \text{ Oe}) \qquad \frac{(\Delta x)_{max}}{(\Delta x)_{min}} = 2.10 \times 10^{12} \quad (3000 \text{ Oe}) \tag{73}$$

If we use our estimate from above

$$(\Delta x)_{min} \sim 3.2 \times 10^{-4} \text{ Angstrom} \tag{74}$$

then we end up with

$$(\Delta x)_{max} \sim 20.0 \text{ cm} \quad (1000 \text{ Oe}) \qquad (\Delta x)_{max} = 6.72 \text{ cm} \quad (3000 \text{ Oe}) \tag{75}$$

These numbers we might compare to the distance associated with the proton velocity for 5 keV (for the x-degree of freedom) at a frequency of ω_a estimated according to

$$\frac{v}{\omega_a} = 20.3 \text{ cm} \quad (1000 \text{ Oe}) \qquad \frac{v}{\omega_a} = 6.75 \text{ cm} \quad (3000 \text{ Oe}) \tag{76}$$

13. Uncertainty Product for the Wave Packet

Of interest is the uncertainty product $\Delta x \Delta p_x$ for the wave packet solution. To proceed, we can use Mathematica to evaluate

$$(\Delta x)^2 = \frac{1}{2\mathrm{Re}\{\alpha_{xx}(t)\}} \qquad (\Delta p_x)^2 = \frac{\hbar^2 |\alpha_{xx}(t)|^2}{2\mathrm{Re}\{\alpha_{xx}(t)\}} \tag{77}$$

For the $\Delta x \Delta p_x$ product we can write

$$\Delta x \Delta p_x = \frac{\hbar}{2} \frac{|\alpha_{xx}(t)|}{\mathrm{Re}\{\alpha_{xx}(t)\}} \tag{78}$$

This can also be written in the form

$$\Delta x \Delta p_x = \frac{\hbar}{2} \frac{1}{\sqrt{1 - \frac{\mathrm{Im}\{\alpha_{xx}(t)\}^2}{|\alpha_{xx}(t)|^2}}} \tag{79}$$

This is of the form

$$\Delta x \Delta p_x = \frac{\hbar}{2} \frac{1}{\sqrt{1 - r_{px}^2(t)}} \tag{80}$$

where

$$r_{px}(t) = \pm \frac{\mathrm{Im}\{\alpha_{xx}(t)\}}{|\alpha_{xx}(t)|} \tag{81}$$

This is seen to be consistent with the Schrodinger-Robertson uncertainty relation. We can write explicitly

$$r_{px}^2(t) = \frac{\sin^2(2\omega_a t)\left(\alpha_{xx}^2(0)\hbar^2 - m^2\omega_a^2\right)^2}{\sin^2(2\omega_a t)\left(\alpha_{xx}^2(0)\hbar^2 - m^2\omega_a^2\right)^2 + 4\alpha_{xx}^2(0)\hbar^2 m^2 \omega_a^2} \tag{82}$$

14. Conclusions

A primary motivation for the construction documented above was to try to help and support Vysotskii and Vysotskyy in their efforts to develop models for LENR reactions. An additional motivation was that we had worked with squeezed wave packet solutions in an academic setting, and we were interested in the application of these solutions to the particle in magnetic field problem proposed by these authors.

As can be seen, it is possible to plug the three-dimensional wave packet solution into the Schrodinger equation for a nonrelativistic particle in a constant z-directed magnetic field and obtain evolution equations from the resulting constraints with modest effort (as long as we require that the evolution in the x and y degrees of freedom match). The spread in position and momentum can be combined to construct a dynamic uncertainty product that has a minimum Schrodinger-Robertson uncertainty. This is consistent with the approach proposed by Dodonov and coworkers for the construction of correlated coherent states.

It is possible to construct states mathematically in which the fluctuation energy greatly exceeds the classical energy. In order for the fluctuation energy to reach 10 keV, the maximum localization is a small fraction of an Angstrom, which is indicative of extreme squeezing. In this case the maximum spread of the wave packet is on the cm scale, which means that it would be very fragile in the sense that interactions with neighboring charged and uncharged particles in the surrounding environment would spoil the wave packet phase and prevent subsequent localization.

Vysotskii and Vysotskyy propose working with squeezed wave packets based on excited state eigenfunctions of the SHO [2]. Due to our unfamiliarity with such states, we examined their construction in the simpler case of a one-dimensional simple harmonic oscillator in the Appendix. This construction was successful, and from the construction the dynamics of the "breathing" is the same as for the more conventional squeezed state solution based on simpler complex Gaussian wave packets. We expect independence of the classical and fluctuation energies for these solutions, and provide an expression for the expectation value of the Hamiltonian which gives the constant fluctuation energy (as expected for a constant potential). The uncertainty principle for this wave packet appears to be a generalization of the Schrodinger-Robertson relation that includes a prefactor of $(2n+1)^2$. For a highly excited eigenfunction, the uncertainty will be very large. The amount of squeezing required to boost the energy of such an eigenfunction at 10 eV to have a fluctuation energy of 10 keV is much less than the amount of squeezing needed to boost the ground state eigenfunction energy (on the order of 10 neV) to 10 keV.

Of interest is how the charged particle gains energy in this kind of scheme. In the event that the dynamics can be modeled making use of a dynamic particle charge, then we can understand the situation relatively simply making use of a one-dimensional analog. When the charge is on, the particle in one dimension sees a parabolic potential, and when the charge is off the particle sees a flat potential. However, in both cases the same squeezed state solution applies. For the basic squeezed state solution we might imagine a particle starting in the ground state of the parabolic well, where it is happy to stay so long as the potential doesn't change. When the potential is set to zero, the wave packet expands with free space dynamics, which also happens to coincide with the evolution of a squeezed state. Later on, when the potential turns back on, the expanded wave packet now extends over a larger region, and will have more energy. The subsequent wave packet dynamics are breathing mode dynamics, since a squeezed state was generated by the free-particle evolution, and we would expect subsequent localization to a width less than that of the ground state. Repeated turning on and off would be expected in this simple model to randomly increase or decrease the squeezing (and wave packet energy), resulting in an interesting non-classical mechanism for the transfer of energy from the dynamics potential to the particle. A classical analog particle in this case would sit at the origin and not notice the changes in the potential. Hence, the fluctuation energy increase in this case is a non-classical effect.

If instead the wave packet started in an excited state eigenfunction of the well, then the subsequent evolution would be qualitatively similar, but faster since much more energy is involved. As long as the potential remains following the initialization of the wave packet, the wave packet would remain localized to within the spread of the excited state eigenfunction. When the potential turns off, the wave packet would expand according to free space dynamics, which would involve a much faster expansion since the eigenfunction is that of a highly excited particle. Now when the potential turns back on, the expansion that occurred results in the wave packet extending to regions where the potential is much larger, which corresponds to net (fluctuation) energy gain. In light of the solution given in the Appendix, the wave packet will undergo generalized squeezing dynamics, which will appear as a "breathing" with the basic shape remaining intact. As before, with random switching on and off of the potential, one would expect that energy would be gained on average through the same basic non-classical energy exchange effect.

A question which remains for future work concerns whether the effect persists in a more sophisticated model in which the proton and electrons are modeled explicitly, making use of multi-particle wave packet solutions. The issue is that if an incoming electron collides with a hydrogen atom to produce ionization, then one would expect a localization of the proton with respect to the electrons at the time of the collision. Intuitively, this localization might be expected to impact the delocalization required for non-classical energy exchange. In the worst case scenario, the processes in the scheme needed to make the charge be time-dependent also destroys the delocalization.

Should this worst case scenario play out, it might still be possible for the non-classical energy exchange proposal of Vysotskii and Vysotskyy [3] to work with a minor modification. Consider instead a situation in which charged particles (protons or other charged nuclei as well as electrons) at very low density travel through a magnetic field that is spatially varying and perhaps time varying. If the plasma density is very low (so that collisions are infrequent) then quantum coherent effects generally have a better chance of being important. At times and in regions where the magnetic field is small, then the wave packet would expand in three dimensions according to free space dynamics. When the particle enters a region with a strong magnetic field, then this is roughly equivalent to when the analog potential turns on, which means squeezing could occur. Now, what is important (as implicit in the arguments of Vysotskii and Vysotskyy) is that the change in the equivalent potential is nonadiabatic. The ionization and recombination proposed by Vysotskii and Vysotskyy are proposed to occur on a very short time scale, which makes them maximally non-adiabatic, and maximizes the energy exchange. Analogously, a moving charged particle encountering magnetic fields that change in strength and orientation can experience non-classical energy exchange if the associated dynamics are non-adiabatic.

Squeezed Wave Packet Based on an Excited SHO Eigenfunction

In this Appendix we consider the construction of a wave packet centered around the origin which is a squeezed version of an excited state eigenfunction of the simple harmonic oscillator. Since the fluctuation degrees of freedom are independent of the position and momentum degrees of freedom, we can focus on the fluctuations with the expectation that it would be straightforward to generalize in the case of a moving wave packet.

Schrodinger Equation and Wave Packet

We consider the time-dependent Schrodinger equation

$$i\hbar \frac{\partial}{\partial t}\psi(x,t) \ = \ -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x,t) + \frac{1}{2}m\omega_0^2 x^2 \psi(x,t) \qquad (.1)$$

and assume a solution of the form

$$\psi(x,t) \ = \ \left(\frac{a(t)}{\alpha_0}\right)^{1/4} e^{-i\Theta(t)} e^{ib(t)x^2/2} \phi_n\left(\sqrt{\frac{a(t)}{\alpha_0}}x\right) \qquad (.2)$$

with

$$\phi_n(x) \ = \ \left(\frac{m\omega_0}{\pi\hbar}\right)^{1/4} \frac{1}{\sqrt{2^n n!}} e^{-m\omega_0 x^2/2\hbar} H_n\left(\sqrt{\frac{m\omega_0}{\hbar}}x\right) \qquad (.3)$$

and

$$\alpha_0 \ = \ \frac{m\omega_0}{\hbar} \qquad (.4)$$

We assume that $a(t)$ and $b(t)$ are real.

Plugging in and Simplifying

We plug in and obtain

$$\hbar\phi_n\frac{d}{dt}\Theta(t)+\frac{1}{4}i\hbar\phi_n\frac{1}{a(t)}\frac{d}{dt}a(t)+\frac{1}{2}i\hbar\sqrt{\frac{a(t)}{\alpha_0}}x\phi_n'\frac{1}{a(t)}\frac{d}{dt}a(t)-\frac{1}{2}\hbar x^2\phi_n\frac{d}{dt}b(t)$$

$$=-\frac{\hbar^2}{2m}\frac{a(t)}{\alpha_0}\phi_n''+\frac{m\omega_0}{2}x^2\phi_n-i\frac{\hbar^2}{2m}b(t)\phi_n+\frac{\hbar^2}{2m}x^2b^2(t)\phi_n-i\frac{\hbar^2}{m}\sqrt{\frac{a(t)}{\alpha_0}}x\phi_n'b(t) \quad (.5)$$

We can eliminate terms proportional to $x\phi_n'$ if we set

$$\hbar\frac{d}{dt}a(t) \;=\; -\frac{2\hbar^2}{m}a(t)b(t) \quad (.6)$$

This results in a simplification according to

$$\hbar\phi_n\frac{d}{dt}\Theta(t)+\frac{1}{4}i\hbar\phi_n\frac{1}{a(t)}\frac{d}{dt}a(t)-\frac{1}{2}\hbar x^2\phi_n\frac{d}{dt}b(t)$$

$$=\;-\frac{\hbar^2}{2m}\frac{a(t)}{\alpha_0}\phi_n''+\frac{m\omega_0}{2}x^2\phi_n-i\frac{\hbar^2}{2m}b(t)\phi_n+\frac{\hbar^2}{2m}x^2b^2(t)\phi_n \quad (.7)$$

We can make use of the evolution equation for $a(t)$ to simplify a bit further and write

$$\hbar\phi_n\frac{d}{dt}\Theta(t)-\frac{1}{2}\hbar x^2\phi_n\frac{d}{dt}b(t) \;=\; -\frac{\hbar^2}{2m}\frac{a(t)}{\alpha_0}\phi_n''+\frac{m\omega_0}{2}x^2\phi_n+\frac{\hbar^2}{2m}x^2b^2(t)\phi_n \quad (.8)$$

We can make use of operators to write

$$x^2\phi_n \;=\; \frac{1}{2}\frac{1}{a(t)}(\hat{a}+\hat{a}^\dagger)^2\phi_n\left(\sqrt{\frac{a(t)}{\alpha_0}}x\right)$$

$$\phi_n' \;=\; \sqrt{\frac{\alpha_0}{2}}(\hat{a}-\hat{a}^\dagger)\phi_n\left(\sqrt{\frac{a(t)}{\alpha_0}}x\right)$$

$$x\phi_n' \;=\; \frac{1}{2}\sqrt{\frac{\alpha_0}{a(t)}}(\hat{a}+\hat{a}^\dagger)(\hat{a}-\hat{a}^\dagger)\phi_n\left(\sqrt{\frac{a(t)}{\alpha_0}}x\right)$$

$$\phi_n'' \;=\; \frac{\alpha_0}{2}(\hat{a}-\hat{a}^\dagger)^2\phi_n\left(\sqrt{\frac{a(t)}{\alpha_0}}x\right) \quad (.9)$$

This leads to

$$\hbar \frac{d}{dt}\Theta(t)\phi_n - \frac{1}{4}\hbar(\hat{a}+\hat{a}^\dagger)^2\phi_n \frac{1}{a(t)}\frac{d}{dt}b(t)$$

$$= -\frac{\hbar^2}{4m}a(t)(\hat{a}-\hat{a}^\dagger)^2\phi_n + \frac{m\omega_0}{4}\frac{1}{a(t)}(\hat{a}+\hat{a}^\dagger)^2\phi_n + \frac{\hbar^2}{4m}\frac{b^2(t)}{a(t)}(\hat{a}+\hat{a}^\dagger)^2\phi_n \tag{.10}$$

We can expand and simplify to obtain

$$\hbar \frac{d}{dt}\Theta(t)\phi_n - \frac{1}{4}\hbar\hat{a}^2\phi_n\frac{1}{a(t)}\frac{d}{dt}b(t) - \frac{1}{4}\hbar(2n+1)\phi_n\frac{1}{a(t)}\frac{d}{dt}b(t) - \frac{1}{4}\hbar(\hat{a}^\dagger)^2\phi_n\frac{1}{a(t)}\frac{d}{dt}b(t)$$

$$\begin{aligned} = \;& -\frac{\hbar^2}{4m}a(t)\hat{a}^2\phi_n + \frac{\hbar^2}{4m}a(t)(2n+1)\phi_n - \frac{\hbar^2}{4m}a(t)(\hat{a}^\dagger)^2\phi_n \\ & + \frac{m\omega_0}{4}\frac{1}{a(t)}\hat{a}^2\phi_n + \frac{m\omega_0}{4}\frac{1}{a(t)}(2n+1)\phi_n + \frac{m\omega_0}{4}\frac{1}{a(t)}(\hat{a}^\dagger)^2\phi_n \\ & + \frac{\hbar^2}{4m}\frac{b^2(t)}{a(t)}\hat{a}^2\phi_n + \frac{\hbar^2}{4m}\frac{b^2(t)}{a(t)}(2n+1)\phi_n + \frac{\hbar^2}{4m}\frac{b^2(t)}{a(t)}(\hat{a}^\dagger)^2\phi_n \end{aligned} \tag{.11}$$

Matching Terms

Matching the $\hat{a}^2\phi_n$ and $(\hat{a}^\dagger)^2\phi_n$ terms leads to the constraint

$$-\frac{1}{4}\hbar\frac{1}{a(t)}\frac{d}{dt}b(t) \;=\; -\frac{\hbar^2}{4m}a(t) + \frac{m\omega_0}{4}\frac{1}{a(t)} + \frac{\hbar^2}{4m}\frac{b^2(t)}{a(t)} \tag{.12}$$

which can be written as

$$\hbar\frac{d}{dt}b(t) \;=\; \frac{\hbar^2}{m}a^2(t) - \frac{\hbar^2}{m}b^2(t) - m\omega_0 \tag{.13}$$

Matching terms proportional to ϕ_n leads to

$$\hbar\frac{d}{dt}\Theta(t) - \frac{1}{4}\hbar(2n+1)\frac{1}{a(t)}\frac{d}{dt}b(t) \;=$$

$$\frac{\hbar^2}{4m}a(t)(2n+1) + \frac{m\omega_0}{4}\frac{1}{a(t)}(2n+1) + \frac{\hbar^2}{4m}\frac{b^2(t)}{a(t)}(2n+1) \tag{.14}$$

This simplifies to

$$
\begin{aligned}
\hbar\frac{d}{dt}\Theta(t) &= (2n+1)\frac{\hbar^2}{4m}a(t) + (2n+1)\frac{m\omega_0}{4}\frac{1}{a(t)} + (2n+1)\frac{\hbar^2}{4m}\frac{b^2(t)}{a(t)} \\
&+ (2n+1)\frac{1}{4}\left(\frac{\hbar^2}{m}a(t) - \frac{\hbar^2}{m}\frac{b^2(t)}{a(t)} - m\omega_0\frac{1}{a(t)}\right) \\
&= (2n+1)\frac{\hbar^2}{2m}a(t)
\end{aligned}
\tag{.15}
$$

Discussion

From the discussion above we see that the proposed wave packet solution satisfies the time-dependent Schrodinger equation. We can summarize the evolution equations that were obtained as

$$\hbar\frac{d}{dt}a(t) = -\frac{2\hbar^2}{m}a(t)b(t) \tag{.16}$$

$$\hbar\frac{d}{dt}b(t) = \frac{\hbar^2}{m}a^2(t) - \frac{\hbar^2}{m}b^2(t) - m\omega_0 \tag{.17}$$

$$\hbar\frac{d}{dt}\Theta(t) = (2n+1)\frac{\hbar^2}{2m}a(t) \tag{.18}$$

It is possible to combine $a(t)$ and $b(t)$ into a single complex variable $\alpha(t)$ according to

$$\alpha(t) \;=\; a(t) - ib(t) \tag{.19}$$

In terms of $\alpha(t)$ the dynamical equations become

$$i\hbar\frac{d}{dt}\alpha(t) = \frac{\hbar^2}{m}\alpha^2(t) - m\omega_0 \tag{.20}$$

$$\hbar\frac{d}{dt}\Theta(t) = (2n+1)\frac{\hbar^2}{2m}\mathrm{Re}\{\alpha(t)\} \tag{.21}$$

The expectation value for the Hamiltonian is

$$
\begin{aligned}
\langle\hat{H}\rangle &= \left\langle -\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\right\rangle + \left\langle \frac{1}{2}m\omega_0^2x^2\right\rangle \\
&= (2n+1)\frac{\hbar\omega_0}{4}\left(\frac{a(t)}{\alpha_0} + \frac{\alpha_0}{a(t)}\right) + (2n+1)\frac{\hbar^2}{4m}\frac{b^2(t)}{a(t)} \\
&= (2n+1)\frac{\hbar^2}{4m}\frac{|\alpha(t)|^2}{\mathrm{Re}\{\alpha(t)\}} + (2n+1)\frac{m\omega_0^2}{4}\frac{1}{\mathrm{Re}\{\alpha(t)\}}
\end{aligned}
\tag{.22}
$$

which is a constant for this model.

To develop an uncertainty principle for this wave packet, we can start with

$$(\Delta x)^2 \;=\; \frac{1}{2}(2n+1)\frac{1}{\mathrm{Re}\{\alpha(t)\}} \qquad (\Delta p)^2 \;=\; \frac{1}{2}(2n+1)\hbar^2\frac{|\alpha(t)|^2}{\mathrm{Re}\{\alpha(t)\}} \tag{.23}$$

We can use these to write

$$(\Delta x)^2(\Delta p)^2 \;=\; (2n+1)^2\frac{\hbar^4}{4}\frac{|\alpha(t)|^2}{\mathrm{Re}\{\alpha(t)\}^2} \;=\; (2n+1)^2\frac{\hbar^4}{4}\frac{1}{1-\frac{\mathrm{Im}\{\alpha(t)\}^2}{|\alpha(t)|^2}} \tag{.24}$$

The conclusion is that

$$\Delta x \Delta p \;=\; (2n+1)\frac{\hbar}{2}\frac{1}{\sqrt{1-r^2(t)}} \tag{.25}$$

with

$$r(t) \;=\; \pm\frac{\mathrm{Im}\{\alpha(t)\}}{|\alpha(t)|} \tag{.26}$$

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