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Research Article

An Examination of the Updated Empirical Data in Support of the Shell Model

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Abstract

This paper is an examination of the updated experimental data, as is currently known in 2020, in support of the shell model and its concepts. The shell model of the nuclear force is considered to be the fundamental and foundational model of the nuclear force – that force which binds the nucleons together in a nucleus. The shell model was developed in the late 1940s, and it is based on the experimentally known nuclear data at that time, data which supported the concept of nuclear shells. Most textbooks, even the current ones, present this experimental data from the 1940s when discussing the validity of the shell model. However, a large amount of nuclear data has since been collected over the past 70 years, and a re-examination of the experimental data in support of the shell model is long overdue.

Keywords: Liquid drop model, Nuclear bonding, Nuclear force, Nuclear stability. Separation energy, Shell model

1. Introduction

The first nuclear shell model was proposed by Dmitry Ivanenko in 1932 soon after the discovery of the neutron [1]. The model was further developed in 1949 by several physicists, most notably Maria Goeppert Mayer, J. Hans D. Jensen, and Eugene Paul Wigner, who shared the 1963 Nobel Prize in physics for the development of the model [2,3]. The nuclear shell model is similar to the concept of the electronic shell structure of atoms, in that it proposes a similar shell-like structure for the protons and neutrons within a nucleus.

When the nuclear shell model was first proposed, there were immediate objections to it, mainly because the nuclear shell model is based on a centrally-located force. However, as inferred by the experimental nuclear binding curve, the nuclear force is not a centrally-located force. As the number of nucleons A increases, a centrally located force would have a parabolically increasing curve for binding energy vs. A, but experimentally, this is not seen. Rather, the nuclear force is commonly referred to as a saturated force, in which the (binding energy/A) vs. A is relatively flat, or "saturated". Another reason for the initial objections to the shell model was due to its underlying differences with the liquid drop model. The shell model is based on the concept that the nucleons within a nucleus move independently,

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and interact with one another only rarely, if at all. Rather the nucleons interact with an energy well, that is used by the Schrödinger equation to determine the nuclear behavior. Quite opposite of this concept, the liquid drop model is based on the concept that there is a collective motion of the nucleus, with a bonding interaction between neighboring nucleons, similar to a drop of liquid.

The shell model includes certain "magic" numbers, numbers that were found by examining the empirical data, comparing it to the liquid drop model, and searching for discrepancies. It was found, from the nuclear data of the 1940s, that there were slight discrepancies in the nuclear behavior when either Z or N is equal to 2, 8, 20, 28, 50, 82, and 126. The nuclear shell model asserts that the explanation for these discrepancies is the existence of nuclear shells.

To get these magic numbers theoretically, a complicated theoretical process is involved. The shell model uses an average potential energy well with a spherically symmetric square well with rounded edges. To this potential, an empirical spin–orbit coupling must be added. These additional variables, which account for geometrical considerations, vibrations, rotational excitations, and pairing properties are used when employing the Schrödinger equation [4]. There are other models that offer more refinements to the shell model, such as the collective model [5,6]. The collective model is described by Bohr as being a generalization of the shell model, in which the nuclear field is no longer considered to be considered constant, but rather to be considered to be a dynamic variable, in that the net nuclear potential undergoes deformations away from a spherically-symmetric square well. Other more recent models for the nuclear force include the nuclear cluster model [7], the residual chromodynamic force model [8] and the electromagnetic model [9].

In this paper, the latest and most updated nuclear databases are used to examine the empirical nuclear data [10] in support of the shell model. This updated data is used to reconstruct the diagrams, graphs, and other empirical evidence in support of the claims of the shell model.

Claim 1. There are incongruities in the binding energy per nucleon when comparing experimental data to the predictions of the semi-empirical formula.

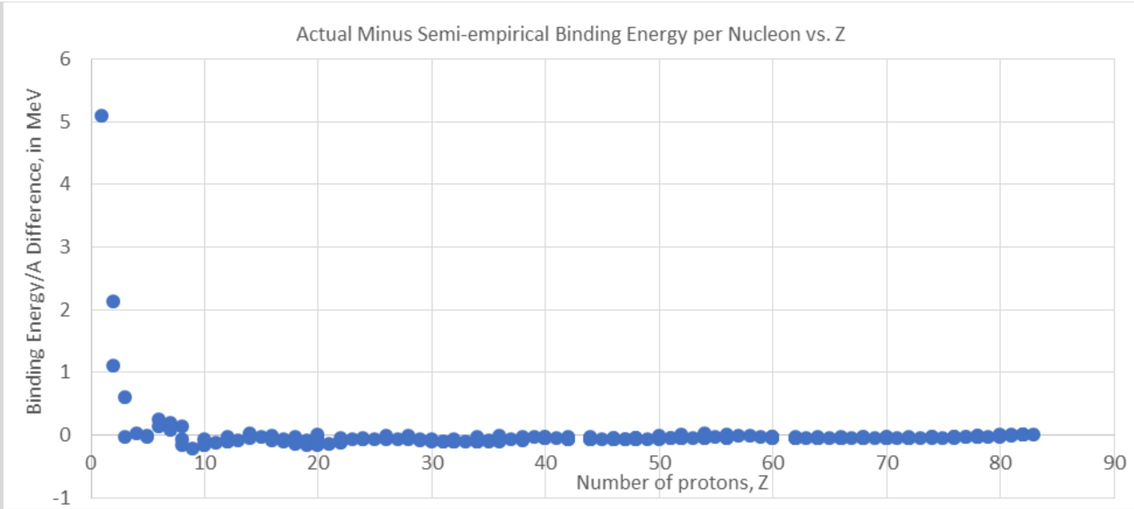
When comparing the binding energy per nucleon to the predicted value from the semi-empirical formula of the liquid drop model, one can see a slight bump at $Z = 28$, 50, 82 and at $N = 28$, 50, 82, and 126. Figure 1(a,b) shows the difference between the semi-empirical formula calculations and the actual binding energy per nucleon, vs. Z and N. As can be seen, this incongruity is small. Usually, the left-hand sides of these graphs are not shown, to hide the very large discrepancies there. These charts shown in Fig. 1(a,b) are generated by inserting the values of Z, N, and A into the semi-empirical equation, and then subtracting the experimental values binding energy for each nuclide. The slight discrepancies seen at the magic numbers seem to be little more than a minor variation, especially when compared to the large discrepancies of the smaller nuclides.

Claim 2a. There are more known isotopes and more stable isotopes, when Z is a magic number.

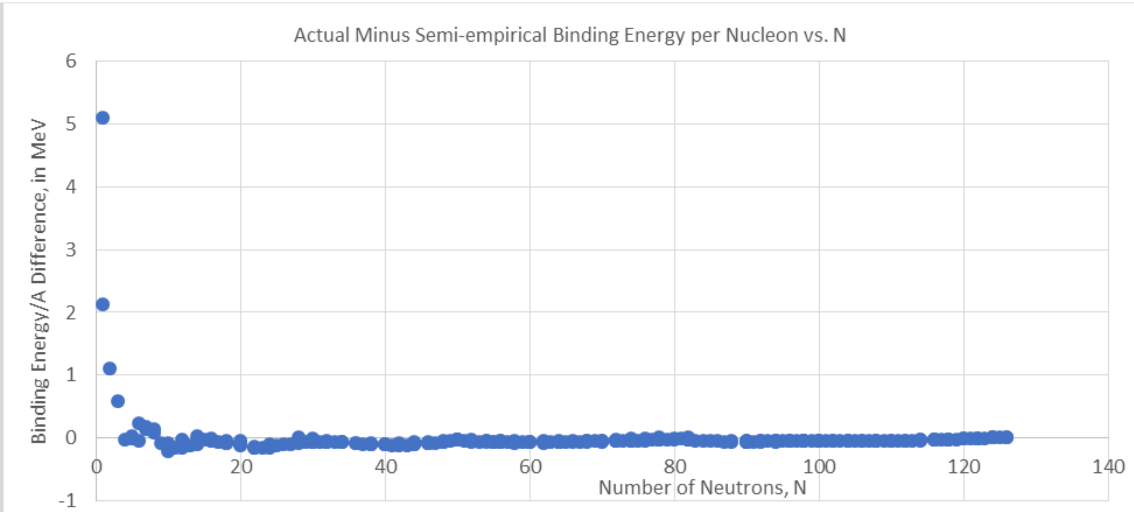
A second claim for evidence of shells is that there are more known isotopes and more stable isotopes when the number of protons Z is equal to a magic number. Shown in Fig. 2(b) is a plot of the number of known isotopes and the number of stable isotopes vs. Z [10]. By looking at the magic numbers of 2, 8, 20, 28, 50, and 82 on the x-axis, and comparing the number of known isotopes (*blue dots*) for that magic number, it can be seen that there are very slight, rather unremarkable, bumps at Z equal to 28 and 50. There are also bumps at 56 and 80, but these are not magic numbers. There are no apparent bumps at any of the other magic numbers. For the number of stable isotopes (*red dots*) shown in Fig. 2(a), a small effect can be seen for $Z = 20$ and 50, but again the other magic numbers do not have any variation in the number of stable isotopes.

For protons, the updated data shows this claim to be true for only two magic numbers, and even then, it is a rather insignificant effect.

Claim 2b. There are more known isotones and more stable isotones, when N is a magic number.



(a)

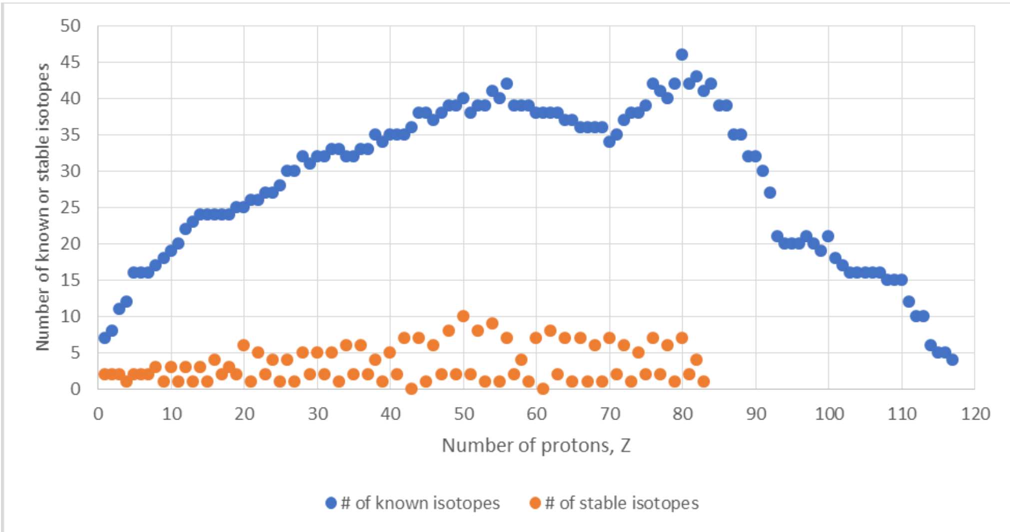


(b)

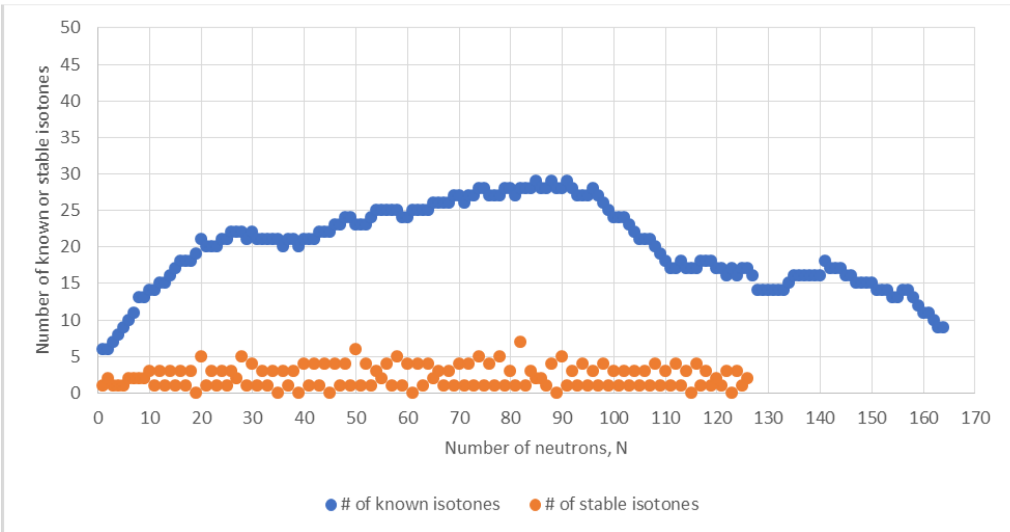
Figure 1. (a) Difference between semi-empirical formula and actual binding energy per nucleon, vs. *Z*. (b) Difference between semi-empirical formula and actual binding energy per nucleon, vs. *N*. .

The second part of this second claim is that there are more known isotones and more stable isotones when the number of neutrons N is equal to a magic number. Shown in Fig. 2(b), on a similar vertical scale as Fig. 2(a) for easier comparison, is a plot of the number of known isotones and the number of stable isotones vs. N [10]. By looking at the magic numbers of 2, 8, 20, 28, 50, and 82 on the x-axis, and comparing the number of known isotones (*blue dots*) for that magic number, it can be seen that there is a very slight bump of one additional isotone at $N = 20$. There are similar slight bumps at 85, 88, 91, and 97, but these are not magic numbers. There are no bumps at any of the other magic numbers. For the number of stable isotones (*red dots*), a small effect can be seen for $Z = 20$, 28, 50, and 82, but again the other magic numbers of 2, 8, and 126 do not have any notable variation in the number of stable isotopes.

Thus similar to protons, the updated data for neutrons shows this claim to be only partially true for only four of the magic numbers. Again, it is only a slight effect, and not what one should consider to be a significant indicator of nuclear shell structure.

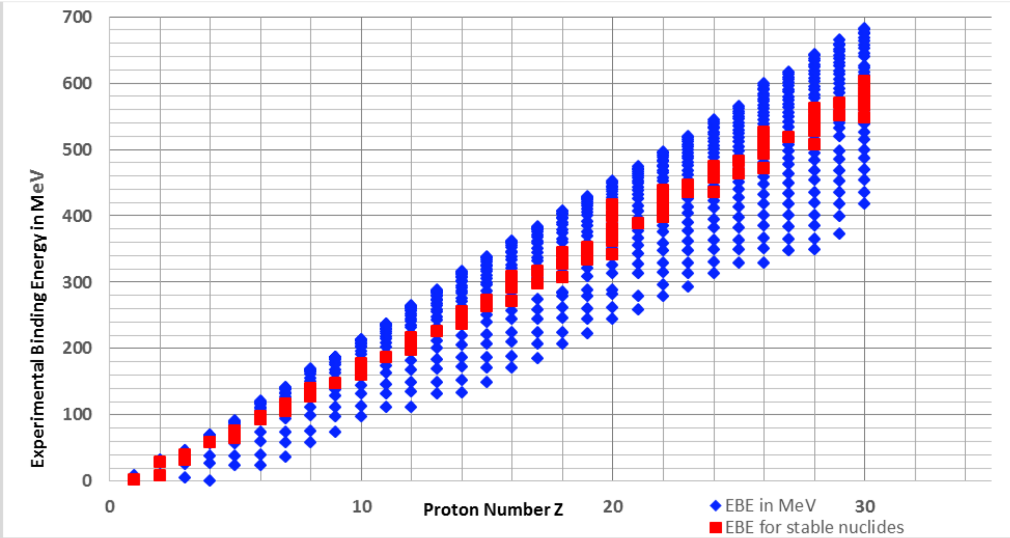


(a)

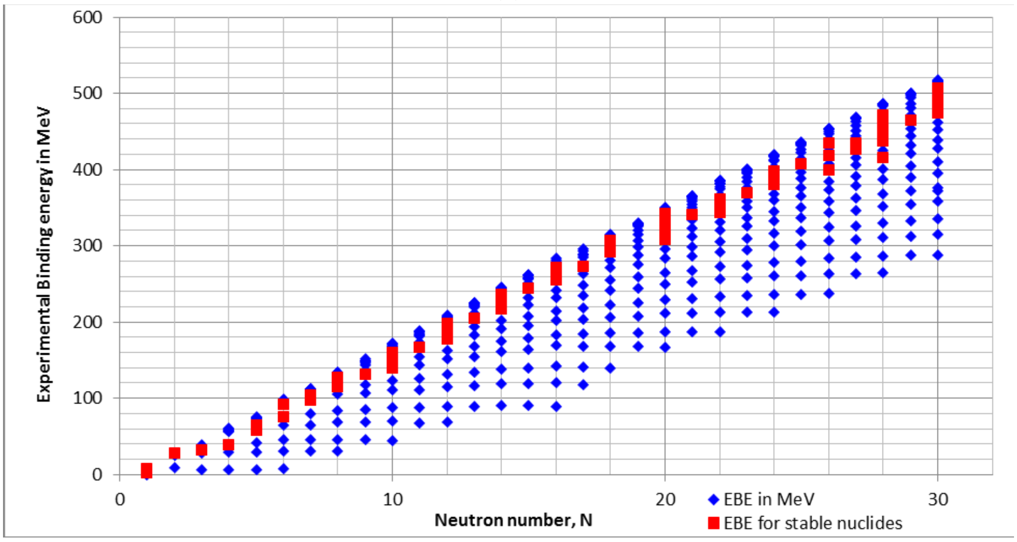


(b)

Figure 2. (a) The number of known and stable isotopes vs. *Z*. (b) The number of known and stable isotones vs. *N*.



(a)



(b)

Figure 3. (a) Experimental binding energy per nucleon vs. *Z*. (b) Experimental binding energy per nucleon vs. *N*.

There is another inconsistency in this second claim of there being more stable nuclides for magic numbers. When the mode of decay of an unstable nuclide is due to beta decay rather than particle decay, then the instability of the nuclide is due to the weak nuclear force. If the shell model is claiming to be able to predict the behaviors of the weak nuclear force, then there are many other more salient behaviors of the weak nuclear force that should be answered that are unrelated to magic numbers or shells. For example, why do nuclides with odd Z tend to have only one or two

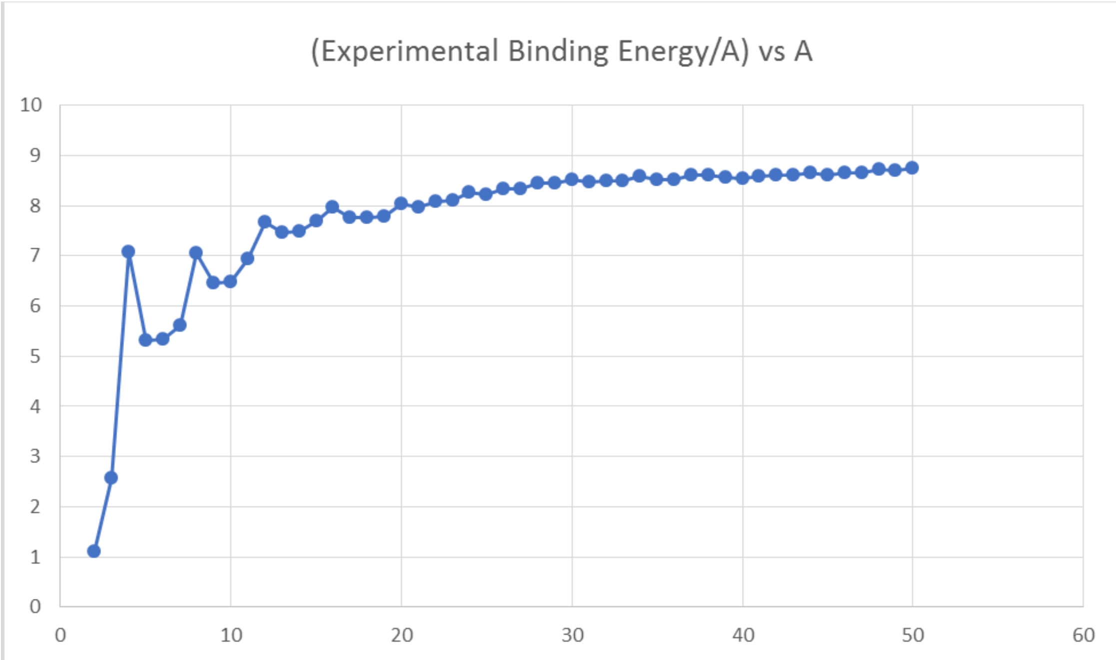


Figure 4. Experimental binding energy per nucleon vs. A for the first 50 stable nuclides.

stable isotopes, and similarly, why do nuclides with the odd N tend to have only zero or one stable isotone? There are many other behaviors of the weak nuclear force that show no correlation to the magic numbers of the shell model. Thus, it would be incorrect to imply that the shell model is a model for the weak nuclear force. Given that, the number of stable of isotopes or isotones for a given Z or N should be considered as being more relevant to the weak nuclear force, rather than being considered as evidence in support of the nuclear shells.

Claim 3. Nuclei with a magic number for Z or N have a higher binding energy than non-magic nuclei.

Another claim of the shell model is that nuclei with either Zor N equal to a magic number have a higher binding energy than non-magic nuclei. Figure 3(a,b) shows the experimental binding energy for all the nuclides, stable and unstable. Figure 3(a) shows the binding energy vs. Z, and Fig. 3(b) shows the binding energy vs. N. As is seen in the figures, there is not a higher binding energy for the magic numbers. The evidence of there being higher binding energy for nuclides with magic numbers at 2, 8, 20 or 28 is not evident in either graph, and going out further to larger values of Z or N also shows no such effect in binding energy.

As seen in Fig. 3(a,b), the nuclides with a magic number of either 2, 8, 20 or 28 are not more tightly bound than the other nuclides near them. For example, all nuclides of Helium have the magic number of $Z = 2$, but other than ^{3}He and ^{4}He they are not stable. Furthermore, ^{3}He is not tightly bound. Helium-5 is extremely unstable, even though it has a magic number for the number of protons. As another example, the nuclides with $N = 8$, such as ^{11}Li, are not more tightly bound nor more stable than the non-magic values of N.

Another related claim to this one suggests that the “doubly magic” nuclides are more stable and more tightly bond, such as ^{4}He and ^{16}O. However, ^{4}He and ^{16}O are not distinct in their binding energy per nucleon, when compared other

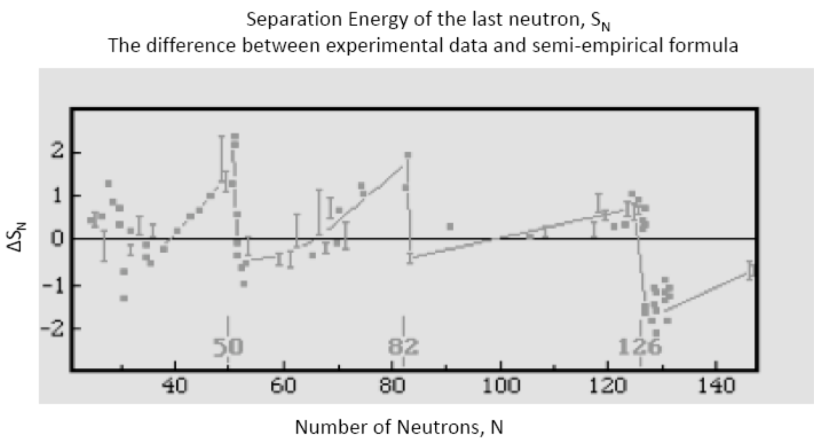


Figure 5. A graph of the binding energy of last neutron vs. *N*, data from the 1940s.

alpha-particle nuclides, such as ^{12}C, ^{20}Ne, ^{24}Mg, ^{28}Si, ^{32}S, ^{36}Ar, or ^{40}Ca. This is shown in Fig. 4 for the Experimental Binding Energy per nucleon vs. A.

All of the alpha-particle nuclides (in which Z is even, and $N = Z$) have a slightly higher binding energy than

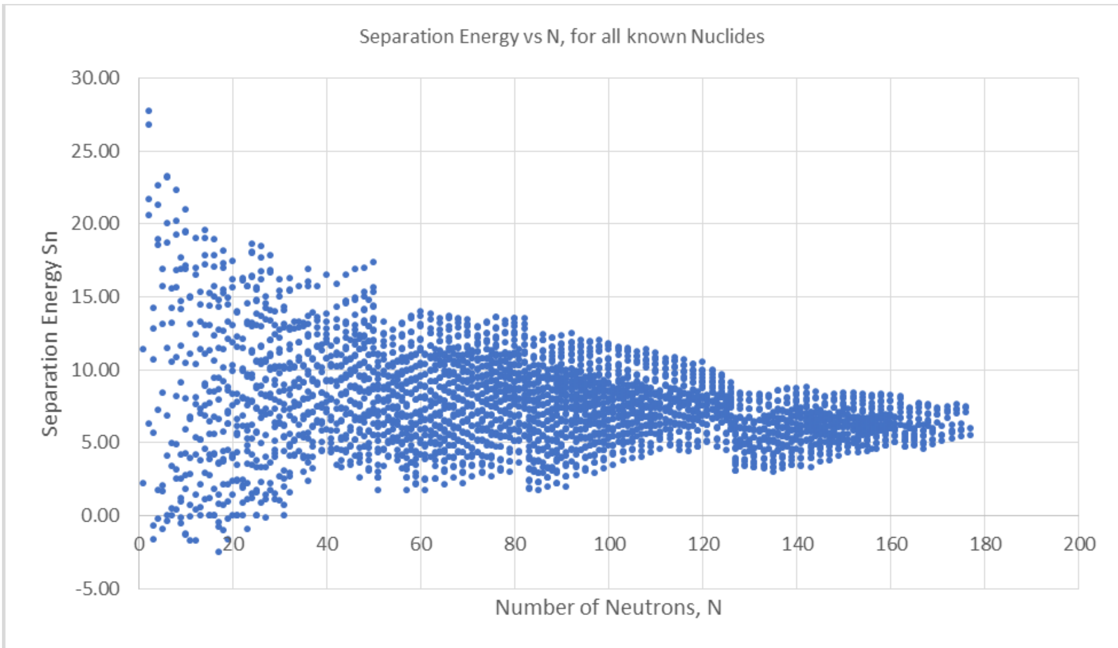


Figure 6. Separation energy, S_n, vs. *N*, for all known nuclides.

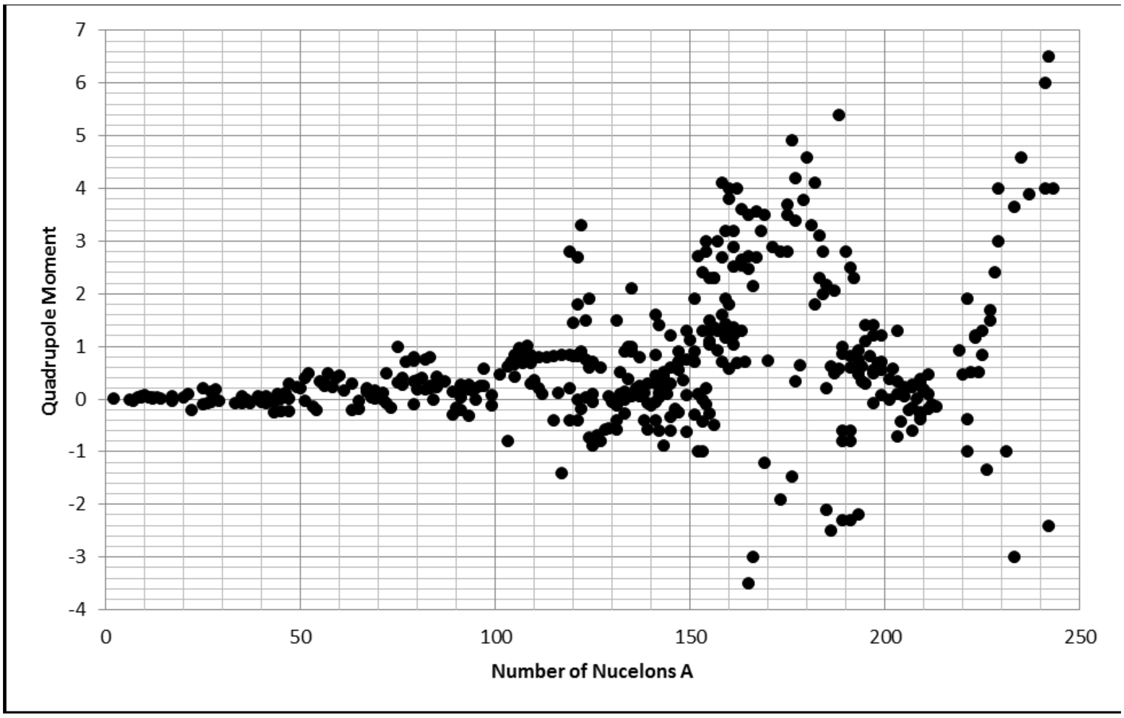


Figure 7. Experimental quadrupole moment. The predicted deformation parameter of the shell model is shown by the blue lines.

the nuclides near them. Thus, this higher binding energy per A at the values of $A = 4$, 8, 12, 16, 20, 24, 28, and 32 supports the cluster model more than the shell model. Also, the claim that doubly magic nuclides are more stable is not valid, as can be seen when examining all the doubly magic nuclides. There are 12 known nuclides that are doubly magic: ^{4}He, ^{10}He, ^{16}O, ^{28}O, ^{40}Ca, ^{48}Ca, ^{48}Ni, ^{56}Ni, ^{78}Ni, ^{100}Sn, ^{132}Sn, and ^{208}Pb. However, only five of them are stable: ^{4}He, ^{16}O, ^{40}Ca, ^{48}Ca, and ^{208}Pb. The remaining seven nuclides are unstable. Thus, being doubly magic does not endow a nuclide with a higher propensity for being more stable.

Claim 4. Neutron separation energies show a distinct zigzag pattern associated with magic numbers, characteristic of shells.

Another claim in support of shell model numbers is an old graph, reproduced in Fig. 5, showing the separation energy vs. neutron number N. Figure 5 is a reproduction of a graph using data from the 1940s. The separation energy, S_{n}, is the energy required to remove one neutron from a nuclide. This graph in Fig. 5 attempts to emphasize the effect of magic numbers. To obtain this graph, the binding energy of the two nuclides A_N and A_{N-1}, are subtracted. Note that these binding energies go up to 1800 MeV for the larger nuclides. Thus the difference between the two nuclides A_N and A_{N-1}, illustrated in this figure, is a very small difference between two much larger numbers. This difference, obtained from the experimental data, is then compared to the predictions of the semi-empirical formula of the liquid drop model, and the resulting difference of the differences, is plotted as shown in Fig. 5. Similar graphs of this separation energy are seen in numerous text books, claiming that this is strong evidence in support of nuclear shells and magic numbers [11–14].

The reason this graph is considered to support the nuclear shell model is due to he zigzag effect seen in this data. For the atomic shell model for electrons, there is a strong zigzag effect seen in the ionization energy of electrons. A plot of the ionization energy of the electrons around an atom vs. the number of electrons displays this zigzag pattern. As the electronic shells are filled around an atom, the ionization energy exhibits a definite and strongly evident zigzag pattern. If nuclear shells existed within a nucleus, similar to the electronic shells around an atom, then this same zigzag effect should be seen as well.

One obvious problem with this graph in Fig. 5 is that it does not include all the nuclides. Nor does it even include, at the very least, all the stable nuclides. There are over 3200 known nuclides that should be plotted on this graph, and yet the data shown in Fig. 5 only shows a smattering of selected points.

Also, the graph in Fig. 5 is the *difference* between the empirical separation energy and what is predicted by using the semi-empirical model, with no justification for doing that. The electronic shell model data for electron ionization energy is simply a plot of the ionization energy; it is not the difference between the actual ionization energy and another theory's predictions. Thus, the experimental data for the nuclear separation energy should show a zigzag pattern, without having to subtract it from another model's theoretical predictions. Another very evident problem with the graph shown in Fig. 5 is that it cuts off at $N < 20$, obscuring the data for the smaller nuclides. When the nuclides from 1 to 20 are included, the resulting spread is extremely large. This large spread is unrelated to magic numbers. Hence, for all these reasons, the graph in Fig. 5 is not a true representation of the separation energy S_n vs. N.

Figure 6 shows all the actual separation energy S_n of all the known nuclides. (All data for S_n have been extracted from [10].) As can be seen by comparing Figs. 5 and 6, there is much difference between what is purported to be proof a shell-like structure in a nucleus in Fig. 5, and what is actually seen in the updated empirical data for neutron separation energy.

In Fig. 6, there are, indeed, discontinuities at $N = 50$, $N = 82$, and $N = 126$. These discontinuities are considerably small, around 1–2 MeV, and they are minor compared to the overall spread, 30 MeV, of the experimental data for S_n that is seen in Fig. 6. Furthermore, these 1–2 MeV discontinuities are insignificant when compared to the overall binding energies (over 1800 MeV) for the larger nuclides. Even more problematic is that no such effect occurs with regard to magic numbers for the separation energy of a proton, S_p, and its absence implies that nuclear shells are not a credible explanation.

There is no question that the small steps in S_n at $N = 50$, 82, and 126 exist. What is in question, however, is whether steps are the result of shells and magic numbers within the nucleus. With regards to the shell model, the term "magic" is very much a misnomer. Physicists know that magic is not really the explanation for nuclear behaviors. Rather, the word "magic" is might be best explained as a short-cut way of saying, "There is a phenomenon occurring at these numbers which we can not yet explain." At some future time physicists may understand why those small steps occurs at $N = 50$, 82, and 125 for the separation energy Sn. However, there is no question that such a future understanding would be an explanation of something other than magic.

Claim 5. There are deformations in the shape of the nuclei, as seen by the electric quadrupole moment, that occur when a nuclei is far away from a magic number.

Another claim of the shell model is that there are deformations in the shape of the nuclei when the nuclei is far away from a magic number. With regard to the concept of a spherical nuclides, it is claimed that these deformations exist in small clusters within the nuclear chart, occurring in small islands far from the magic numbers. The experimental data for the electric nuclear quadrupole moments, which is directly correlated to the nuclear deformations, shows that this claim is not valid. If the shell model were correct, all of these quadrupole moments, seen in the Fig. 7, should be less than what is indicated by the blue line. (All data extracted from [10,15].)

As can be seen in Fig. 7, there are large deformations in the shape of the nuclides, much larger than can be predicted or explained by the shell model. These deformations exist for the majority of nuclides, not just for small islands far from the magic numbers. Hence the claim that the data for the electric nuclear quadrupole moment support the shell model is not supported by the updated empirical data; rather it is quite contradictory to the shell model. Most of the nuclides are far above the blue lines, the maximum quadrupole moment predicted by the shell model. (The collective model is able to reproduce the large quadrupole moments by adding two to three empirically selected variables for each nuclide. In the collective model, the potential energy well of the Schrödinger equation is not spherically symmetric, but rather it is ellipsoidal, to better match the experimentally observed quadrupole moments.)

2. Discussion

The updated experimental data for nuclides indicate that certain magic numbers may not be as cogent as previously assumed. This is an unanticipated result, but one which is difficult to deny when examining the updated experimental data. Thus, upon reexamination of updated data, the shell model appears to be less of a fundamental model to explain nuclear behavior than was previously believed in the 1940s. Experimental evidence in support of the shell model has diminished in the test of time.

3. Conclusion

It would be misleading for anyone to claim that the shell model is a foundational model, or that nuclear structure is based on nuclear shells, with strong empirical data to support this concept. To be more precise, the nuclear shell model is substantiated only by weak and/or insignificant experimental data. Also, considering that the shell model requires the inclusion of other theoretical models, such as the collective model, to more accurately predict experimental behavior, the shell model seems to be obsolete, both empirically and theoretically. It is suggested that the shell model should only be taught in the context of its historical interest, but it should not be taught as an underlying foundational model of nuclear structure.

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